

Calculation Policy

Addition

Addition - commonly seen in EYFS

Vocabulary

Concrete representations

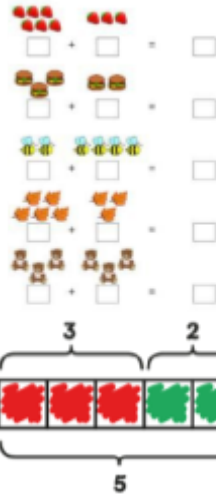


Use a range of different objects to physically manipulate, group and regroup.



Use maths resources such as Numicon, multilink, counters etc to support understanding.

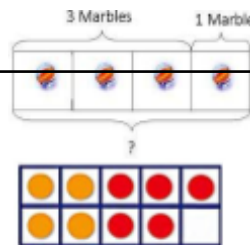
Pictorial representations



Pictures of two groups to enable counting to find the total.

Bar model using visuals, pictures, symbols or colours.

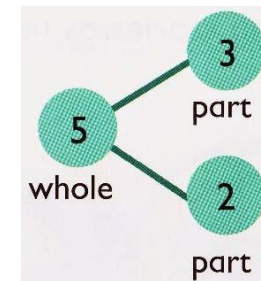
Use visuals such as ten frames, part-part-whole and addition mats with pictures /symbols.



Abstract representations

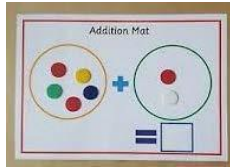
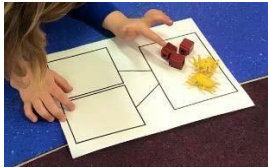
Using symbols and numbers to form calculations.

$$5 + 2 = 7$$

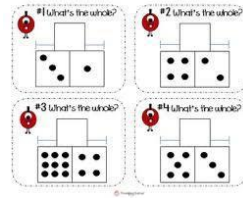
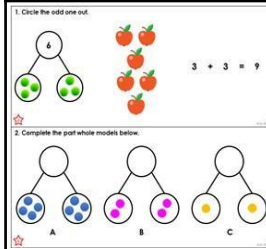


2	3	4	5
5	4	3	3
5	5	6	4

* No expectation that number sentences/ calculations are recorded.



Use visual supports such as 10 frames, part-part-whole diagrams, addition mats with objects.



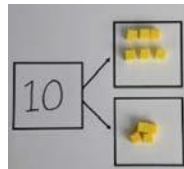
Addition - commonly seen in Year 1

Vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole

Concrete representations

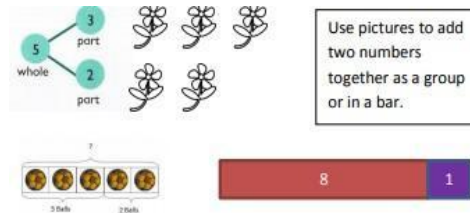


Use cubes, Numicon or other objects to add two numbers together as a group or in a bar.

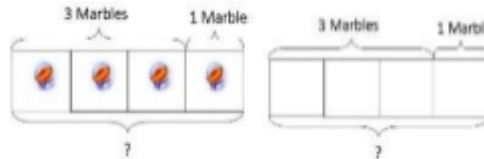


Use part-part-whole model

Pictorial representations



Part-part-whole and Bar Models used to support problem solving involving addition, with concrete and pictorial representations being used alongside each other.



Pictorial (concrete)

Abstract

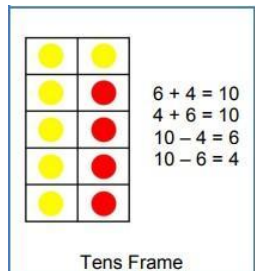
Abstract representations



Use the part-part whole diagram as shown above to move into the abstract.

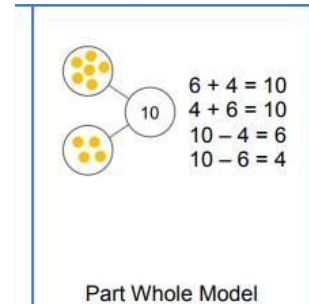
$$4 + 3 = 7$$

$$10 = 6 + 4$$

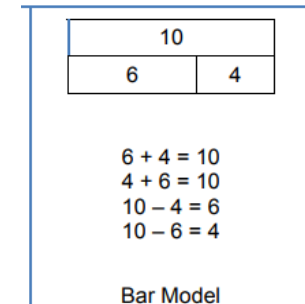


Tens Frame

Use counters, Numicon or other objects, e.g. counters in egg boxes to support understanding.



Part Whole Model



Bar Model

Bar model and part-part-whole used

		alongside abstract.
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Start with the higher number on the bead string/Numicon/tens frame etc and then add the lower number by counting on (1 by 1) to find the answer.	Start at the higher number on the number line and count on in ones. $12 + 5 = 17$	$5 + 12 = 17$ $17 = 12 + 5$ Place the higher number in your head and add on the lower number to find your answer.
$6 + 5 = 11$ Starting with the higher number, use the smaller number to count on to make 10.	$3 + 9 =$ Use pictures or a number line. Regroup or Partition the lower number using the part-part- whole model to make 10. Starting at the higher number on the number line, count on in ones, in jumps to the next 'friendly'; (multiple of 10)	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10? How many more do I add on now?

	number and beyond or in one jump.	
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Addition - commonly seen in Year 2

Vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

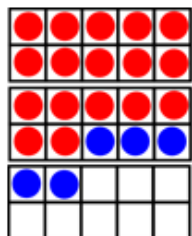
Concrete representations

$$4 + 7 + 6 = 17$$

Put 4 and 6 together to make 10. Add on 7.



Following on from making 10, make 10 with 2 of the digits (where possible) then add on the 3rd digit.



$$17 + 5 = 22$$

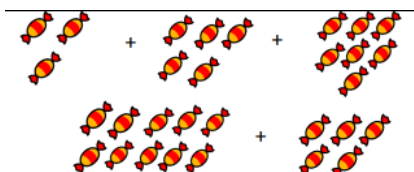
Use ten frame to make 'magic ten'

Children explore the pattern.

$$17 + 5 = 22$$

$$27 + 5 = 32$$

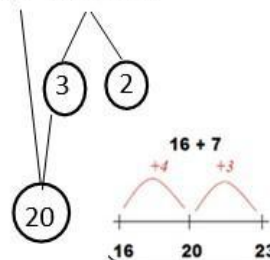
Pictorial representations



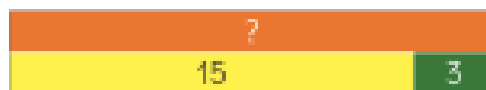
Add together three groups of objects. Draw a picture to recombine the groups to make 10.

$$17 + 5 = 22$$

Use part part whole and number line to model.



Bar Model



Abstract representations

$$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ = 17 \end{array}$$

Combine the two numbers that make 10 and then add on the remainder.

$$17 + 5 = 22$$

Explore related facts

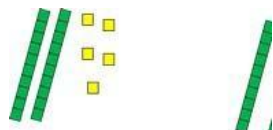
$$17 + 5 = 22$$

$$5 + 17 = 22$$

$$22 - 17 = 5$$

$$22 - 5 = 17$$

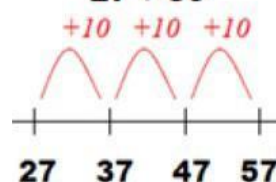
22	
17	5



$$25 + 10 = 35$$

Explore how the ones digit/value does not

$$27 + 30$$



$$27 + 10 = 37$$

$$27 + 20 = 47$$

$$27 + \square = 57$$

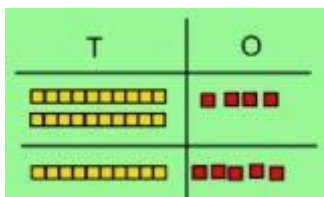
change.	Dienes or other resources may be used alongside the number line. Calculation shown alongside the number line to make links.	
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24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. T O Numicon may also be used by learners if needed.	T O Moving away from practical resources to symbols such as drawn counters to help them to solve calculations. +20 +5 Or +20 +3 +2 47 67 72 47 67 70 72 Use number line and bridge ten using part whole if necessary. Numicon, Dienes etc may be used above the number line if needed. The calculation shown alongside the number line to see the connection. <table><tr><th>Model</th><th>Calculation</th></tr><tr><td></td><td></td></tr></table> The <u>Bar Model</u> used to support problem solving moving onto the generalisation that $b+c = a$. Abstract representation alongside the pictorial to support where necessary.	Model	Calculation			Partitioning: 25 + 47 20 + 5 40 + 7 20 + 40 = 60 5 + 7 =12 60 + 12 = 72 Recording addition in columns supports place value and prepares for formal written methods with larger numbers. Toward the end of the year, learners move towards more formal recording using partitioning method: 40 + 7 30 + 5 70 + 12
Model	Calculation					

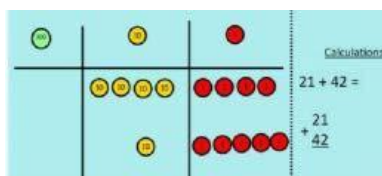
Addition - commonly seen in Year 3

Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Concrete representations



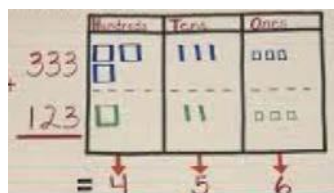
Using manipulatives (Dienes, Numicon, counters etc), learners line up hundreds, tens and ones.



Learners should be secure with using PV counters before moving onto pictorial.

The calculation shown alongside the model used to see the connection.

Pictorial representations



Learners draw in a PV frame, the manipulatives they are using.

Secure knowledge of representation with the PV columns.

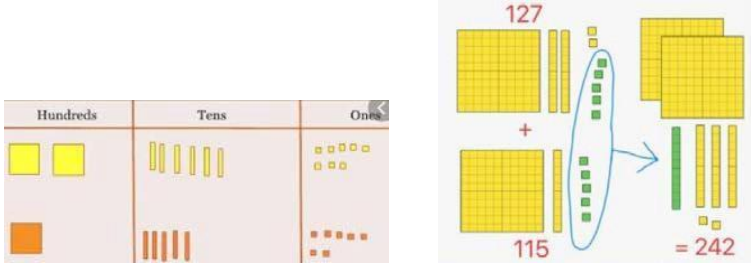
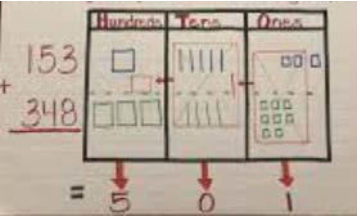
The calculation will be shown alongside the model to see the connection

Abstract representations

$$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$$

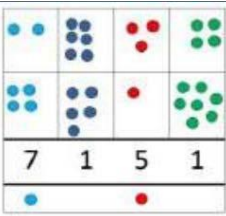
Learners to move towards recording more formally.

Some learners may need to use the expanded method (see below).

 Exchange ten ones for a ten. Model using Dienes, Numicon and place value counters.	 Children can draw a representation of the grid to further support their understanding, carrying the ten <i>underneath</i> the line.	$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ Learners begin with the abstract expanded form. When confident, learners move on to the formal method. $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$
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Addition - commonly seen in Year 4

Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Concrete representations	Pictorial representations	Abstract representations
		



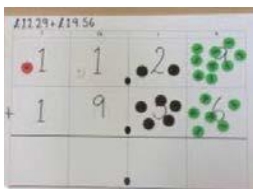
Learners continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.

The calculation shown alongside the manipulative to show the links.

Learners draw a pictorial representation of the columns and place value counters to make links.

$$\begin{array}{r} 3517 \\ + 396 \\ \hline 3913 \end{array}$$

Continue from previous work to carry hundreds as well as tens.



$$\begin{array}{r} £23.59 \\ + £7.55 \\ \hline £31.14 \end{array}$$

As learners move on, introduce decimals with the same number of decimal places and different. Money used here.

Introduce decimal place value counters and model exchange for addition.	$2.37 + 81.79$	
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Addition - commonly seen in Year 5/6

Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal, decimal point

Concrete representations	Pictorial representations	Abstract representations
See Year 4	See Year 4	Abstract supported by a pictorial or concrete if needed.
See Year 4	See Year 4	

		Use zeros for place holders.
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Subtraction

Subtraction - commonly seen in EYFS

Vocabulary subtract, take away, equal, less, fewer, minus, leaves.

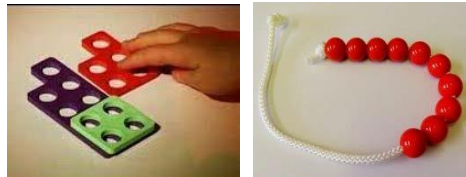
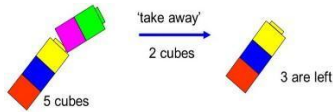
Concrete representations

Pictorial representations

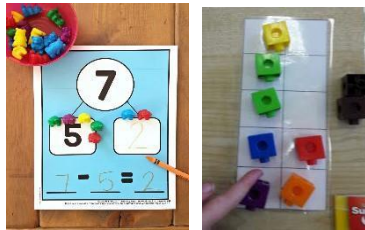
Abstract representations



Use toys and other objects for children to physically manipulate, group/regroup.



Use specific maths resources such as unifix, Numicon, bead strings etc.



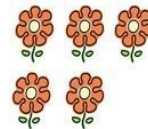
Use visual supports such as ten frames, part-part-whole and subtraction mats, with the objects and resources that can be manipulated.



$$6 - 4 =$$



$$3 - 1 =$$

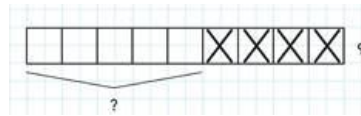
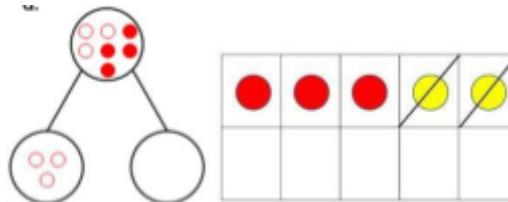


$$5 - 3 =$$



$$7 - 2 =$$

A group of pictures for children to cross out or cover quantities to support subtraction.



Use visual supports such as ten frames, part-part-whole and bar model with pictures/icons.

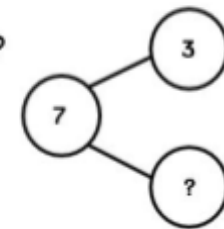
A focus on symbols and numbers to form a calculation.



$$10 - 6 = 4$$

3	?
7	

$$7 - 3 = ?$$

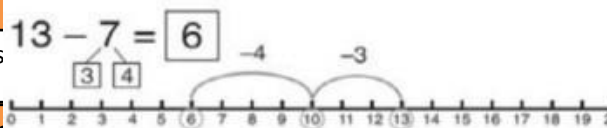
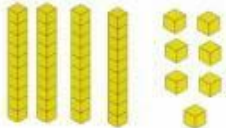
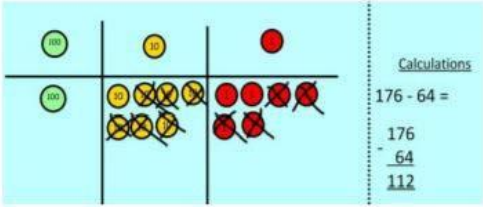
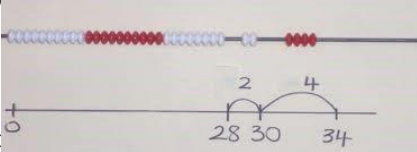
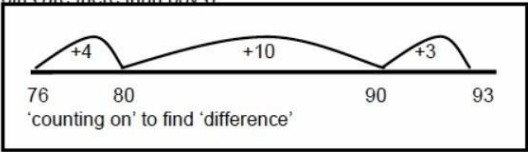
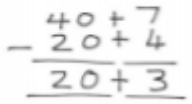
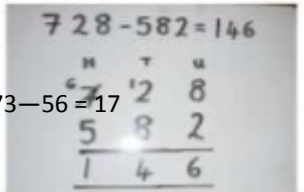


* No expectation that number sentences/ calculations are recorded.

Subtraction- commonly seen in Year 1

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is

Concrete representations	Pictorial representations	Abstract representations
Compare objects and amounts Use p to sh to sh 7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 5 Pencils 3 Erasers Lay objects to represent bar model. 4—2 = 2 4—2 = 2 Start with the counters/ ten counting on 13 - 4 Use counter backwards. 10—6 = 4 COUNT ON INSTEAD???	Pictorial representations +6 Count on to find the difference. Comparison Bar Models Draw bars to find the difference between 2 numbers. Use a pictorial representation of objects to show the part-part-whole model Count back on a number line or track Start at the higher number and count back the lower number Use a pictorial representation of objects to show the part-part-whole model	Abstract representations Harry has 14 sweets and her brother has 8. How many more does Harry have than her brother? 7—4 = 3 16—9 = 7 Put 12 in your head, count back 3. What number are you at? 10 5 Move to using numbers within the part whole model.

14 - 9 =  Vo mu	Subtraction – commonly seen in Year 2 Abstract representations $13 - 7 = 6$  Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	$16 - 9 =$ ss than, most, least count back, how many left, how many do we take off to reach the next 10? How many do we have left to take off? Abstract representations
Vo how Make 14 on the ten frame. Take away the four first to make 10 and then take 34 - 13 = 21 the answer  Use Dienes to show how to partition  47 - 32 C  used. Use base 10 or Numicon to model	tract, leaves as units Abstract representations  43 - 21 = 22  Children a before m and cross off.	ss than, most, least count back, how many left, Recording subtraction in columns supporting place value and, preparing for formal written methods with higher numbers. Abstract representations Toward the end of the year, learners move to more formal methods. Begin with the expanded form then moving on to the formal method. e.g. 53-24=77 50 and 3 -20 and 4 70 and 7
The calculation will be shown alongside the model chosen  Begin with value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange. Use a bead bar or bead strings to model counting to the next ten and the rest.	 Use a number line to count on to next ten and then the rest.	$47 - 24 = 23$  $73 - 56 = 17$ 

Column method (using base 10 and having to exchange)

45-26



- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

$$\begin{array}{r} 45 \\ -29 \\ \hline 16 \end{array}$$

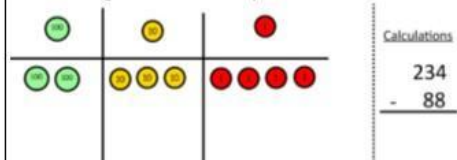
Tens | Ones

10 + 6 = 16

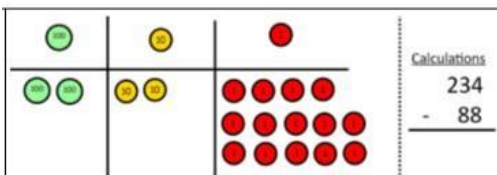
When confident, learners can find their own way to record the exchange/regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

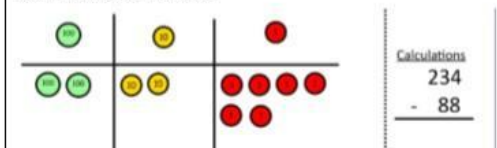
Make the larger number with the place value counters



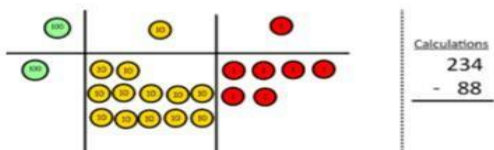
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



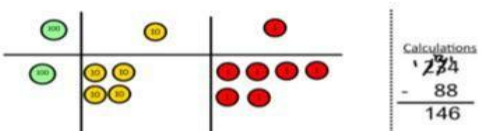
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



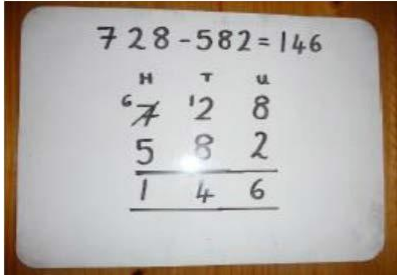
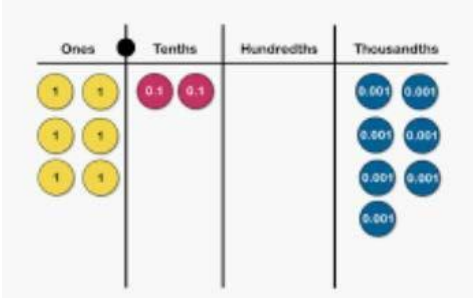
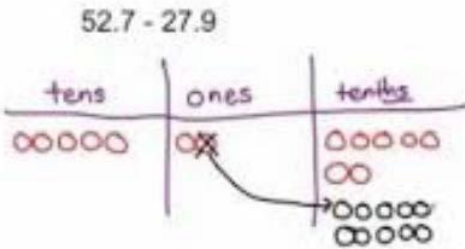
Now I can take away eight tens and complete my subtraction



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Subtraction – commonly seen in Year 4

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Concrete representations	Pictorial representations	Abstract representations
Model process of exchange using Numicon, base ten and then move to using place value counters. Use the phrase 'take and make' for exchange - see Y3 The calculation will be shown alongside the model chosen to show the links.	Learners to draw place value counters and show their exchange — see Y3 The calculation will be shown alongside the model chosen to show the links.	 This will lead to an understanding of subtracting any number including decimals.
Learners to be encouraged to use counters to represent numbers and take counters away to subtract. 	 When confident, learners can find their own way to record the exchange/regrouping.	

Adding & Subtracting Decimals...

Rule 1: line 'em up! Place Value Matters!

$$\begin{array}{r} 1.4 \\ + 6.75 \\ \hline \end{array}$$

$$\begin{array}{r} 15.1 \\ - 7.95 \\ \hline \end{array}$$

Rule 2: drop it down! No decimal changes the value!

$$\begin{array}{r} 1.4 \\ + 6.75 \\ \hline \end{array}$$

$$\begin{array}{r} 15.1 \\ - 7.95 \\ \hline \end{array}$$

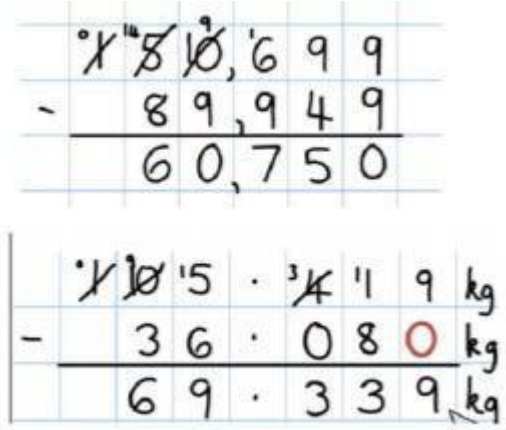
Rule 3: fill 'em in! Think: Does it make sense?

$$\begin{array}{r} 1.40 \\ + 6.75 \\ \hline \end{array}$$

$$\begin{array}{r} 15.10 \\ - 7.95 \\ \hline \end{array}$$

Subtraction – commonly seen in Year 5/6

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units

Concrete representations	Pictorial representations	Abstract representations
See Year 4	See Year 4	 Handwritten subtraction problems on a grid: $\begin{array}{r} \cancel{8}9,949 \\ - \quad \quad \quad \cancel{2}8,199 \\ \hline 60,750 \end{array}$ $\begin{array}{r} \cancel{1}5.36 \text{ kg} \\ - \quad \quad \quad \cancel{8}.03 \text{ kg} \\ \hline 6.93 \text{ kg} \end{array}$

Multiplication

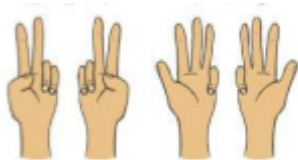
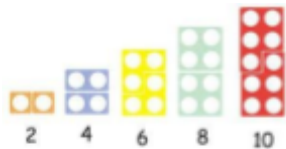
Multiplication - commonly seen in EYFS

Vocabulary

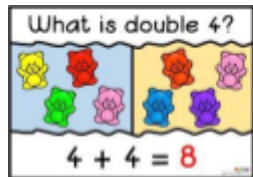
Concrete representations



Using a variety of maths resources and objects, learners make 2 equal groups.



Pictorial representations



Use pictures, icons and symbols that encourage learners to explore the concept of doubling as adding two equal groups.

Abstract representations

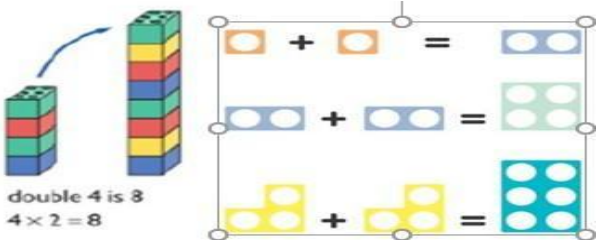
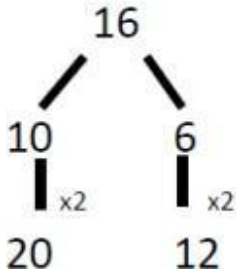
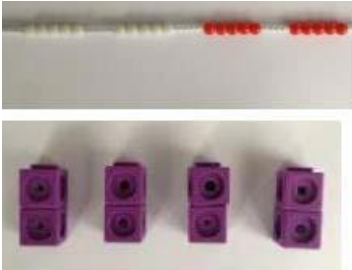
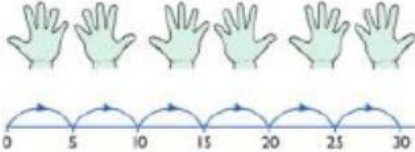
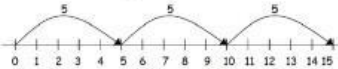
$1+1=$	$7+7=$
$2+2=$	$8+8=$
$3+3=$	$9+9=$
$4+4=$	$10+10=$
$5+5=$	$11+11=$
$6+6=$	$12+12=$

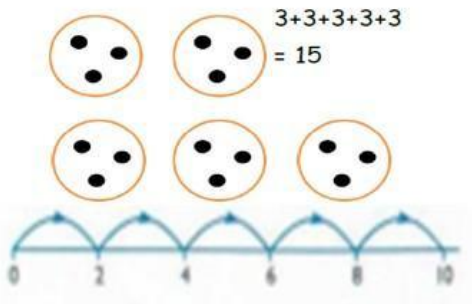
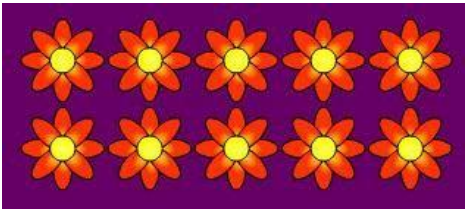
Addition calculations to model adding two equal groups.

Learners explore physical and real-life examples to explore the concept of doubling as adding two equal groups.		
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Multiplication – commonly seen in Year 1

Vocabulary Groups of, lots of, times, array, altogether, multiply, double, equal

Concrete	Pictorial	Abstract
 double 4 is 8 $4 \times 2 = 8$ Learners use practical resources, e.g. cubes and Numicon, to demonstrate doubling.	Double 4 is 8  Learners draw pictures to illustrate doubling.	 Partition a number and then double each part before recombining it back together.
 Calculate multiples using concrete objects and maths resources.	 Calculate multiples using a number line.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30
 Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  $5 + 5 + 5 = 15$	Write addition sentences to describe objects and pictures.  $2 + 2 + 2 + 2 + 2 = 10$

	Use pictures, icons or symbols including number lines to solve problems. There are 3 sweets in one bag. How many sweets in 5 bags?  The diagram illustrates the problem using two methods. At the top, five circles represent bags, each containing three dots representing sweets. To the right of the bags, the equation $3+3+3+3+3 = 15$ is written. Below the bags, a number line from 0 to 10 is shown with five blue curved arrows, each starting at an even number (0, 2, 4, 6, 8) and ending at the next odd number (3, 5, 7, 9, 11), representing jumps of 3.	
Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.  An array of ten orange flowers with yellow centers, arranged in two rows of five, used to illustrate multiplication problems.	 A photograph of two young children, a girl and a boy, standing in a classroom and holding up large sheets of paper displaying their multiplication tables.	$3 \times 2 = 6$ $2 \times 5 = 10$

Multiplication- commonly seen in Year 2

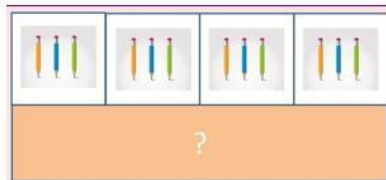
Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.

Concrete representations

Count the groups during skip counting, Use bar models.

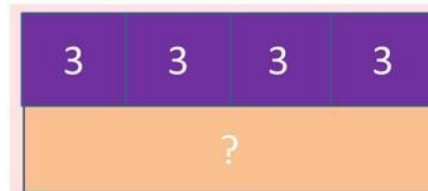
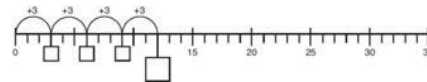


$$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$$



Pictorial representations

Number lines, bar models, counting sticks etc used to represent counting in multiples.



Abstract representations

Count in multiples of a number aloud.

Write sequences with multiples of numbers.

0, 2, 4, 6, 8, 10

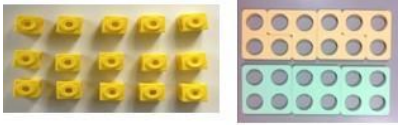
0, 3, 6, 9, 12, 15

0, 5, 10, 15, 20, 25, 30

$$4 \times 3 = \boxed{}$$

Commutativity

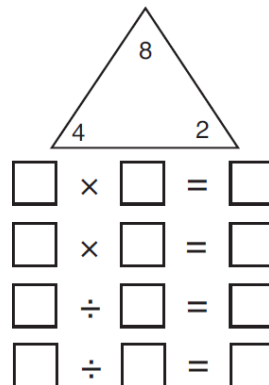
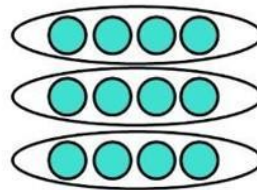
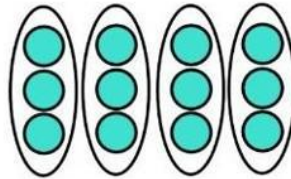
Create arrays using counters, cubes, Numicon.



Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.



Use representations of arrays to show different calculations and explore commutativity.



$$12 = 3 \times 4$$

$$12 = 4 \times 3$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

$$8 \div 2 = 4$$

$$8 \div 4 = 2$$

$$8 = 2 \times 4$$

$$8 = 4 \times 2$$

$$2 = 8 \div 4$$

$$4 = 8 \div 2$$

Show all 8 related fact family sentences.

Multiplication - commonly seen in Year 3

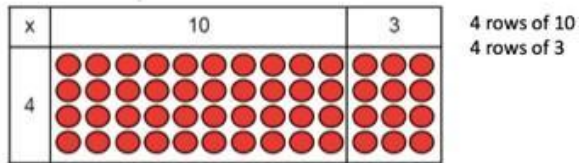
Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up

Concrete representations

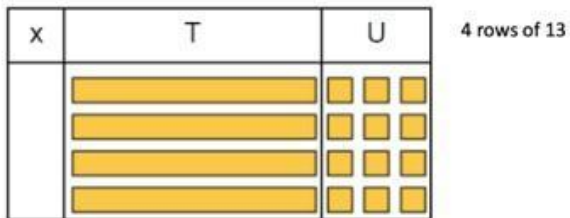
Pictorial representations

Abstract representations

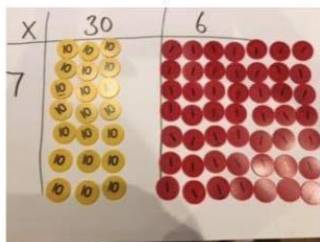
Show the link with arrays to first introduce the grid method.



Move on to using Base 10 to move towards a more compact method.



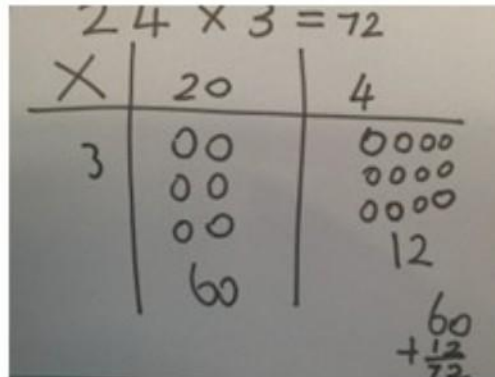
Move on to place value counters to show how we are finding groups of a number.



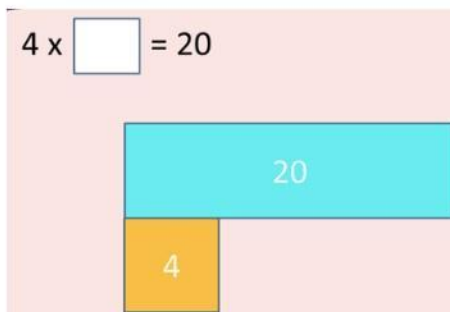
Add up each column, starting with the ones making any exchanges needed.

The calculation will be shown alongside the model chosen to show the links.

Learners can represent their work with place value counters and draw the colours (or just use circles) to show different amounts to show their thinking.

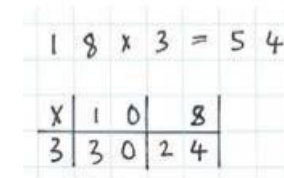


Bar model are used to explore missing numbers



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

TO x O



Children to add up each column to find the answer.

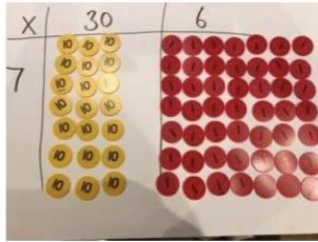
Multiplication - commonly seen in Year 4

Vocabulary Groups of, lots of, times, array, altogether, multiply, Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive

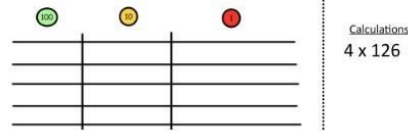
Concrete representations

Pictorial representations

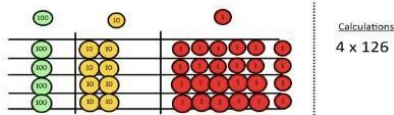
Abstract representations



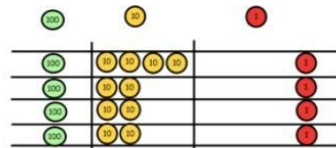
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



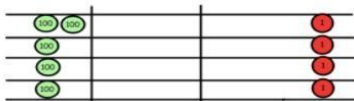
Fill each row with 126.



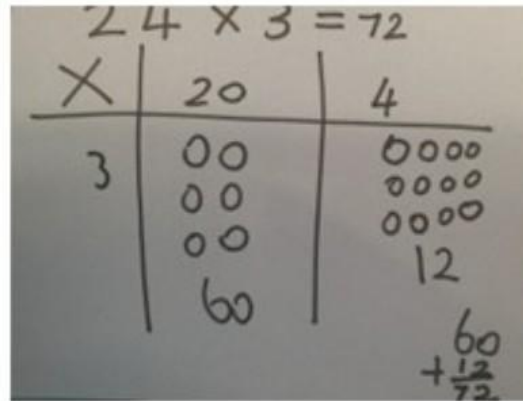
Add up each column, starting with the ones making any exchanges needed.



Then you have your answer.



Learners can represent their jottings with place value counters in a way that they understand. Can draw coloured or plain counters in different columns to show thinking.

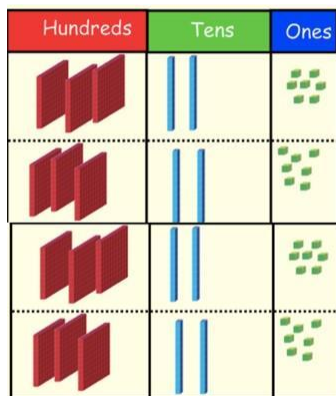


HTO x O

1	3	5	x	5	=	6	7	5
x	1	0	0	3	0	5		
5	5	0	0	1	5	0	2	5

Learners to add up each column to find the answer.

Learners continue to be supported by place value counters at this stage of multiplication. Initially, this is done with no regrouping.



It is important at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside

This grid method can be used to show how this relates to a formal written method.

$$\begin{array}{r} \times \quad 100 \quad 20 \quad 4 \\ 5 \quad 500 \quad 100 \quad 20 \end{array}$$

Bar modelling: A bar divided into 8 equal segments, each labeled 59. Below the bar is a question mark.

Calculation:

$$\begin{aligned} &= 8 \times 59 \\ &8 \times 60 - 8 \\ &8 \times 6 = 48 \\ &8 \times 60 = 480 \\ &480 - 8 = 472 \end{aligned}$$

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

1	2	4	$\times 5 =$
1	2	4	
$\times$		5	
	2	0	(4 $\times$ 5)
	1	0	(20 $\times$ 5)
	5	0	(100 $\times$ 5)
	6	2	0

This may lead to a compact method.

Multiplication – commonly seen in Year 5

Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed.

Concrete representations

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$

Hundreds	Tens	Ones
		
		
		
		

It is important at this stage that they always multiply the ones first.

The corresponding long multiplication is modelled alongside

Pictorial representations

x	300	20	7
4	1200	80	28



Abstract representations

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 1308 \\ \hline \end{array}$$

This may lead to a compact method.

Manipulatives used with the corresponding long multiplication modelled alongside.

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

$$24 \times 16 = 384$$

x	20	4
10200	40	
6120	24	



$$\begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ 240 \\ \hline 384 \end{array}$$

24 x 6 on the first row.

(6 x 4 = 24, carrying the 2 for the 20, then 6 x 2)

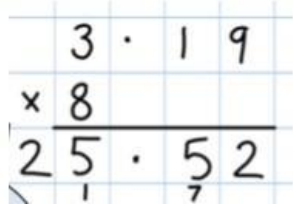
24 x 10 on the second row. Show multiplying by 10 by putting zero in the units first.

$$\begin{array}{r} 1234 \\ \times 16 \\ \hline 7404 \\ 12340 \\ \hline 19744 \end{array}$$

(1234 x 6)
(1234 x 10)

Multiplication- commonly seen in Year 6

Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed.

Concrete representations	Pictorial representations	Abstract representations
		Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Division

Division - commonly seen in EYFS

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array, half, same

Concrete representations

Pictorial representations

Abstract representations



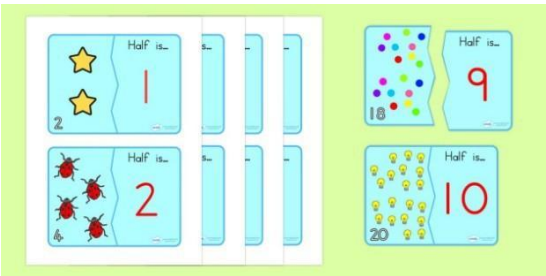
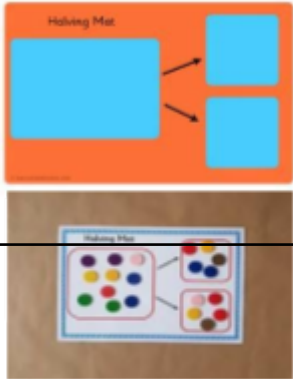
Cut objects, food or shapes in half.



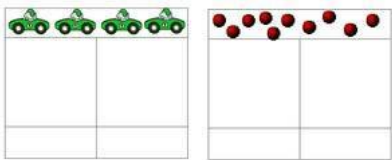
Counting and other maths resources for children to share into two equal groups.

Use visual supports such as halving mats and part-part-whole, with the physical objects and resources that can be manipulated.

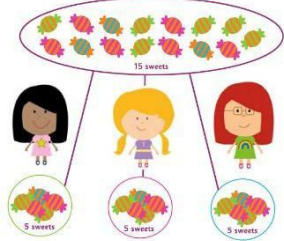
Counting and other maths resources for children to explore sharing between 3 or more.



Use pictures, symbols and icons that show concept of halving in relation to subitising, addition and subtraction knowledge. E.g. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.



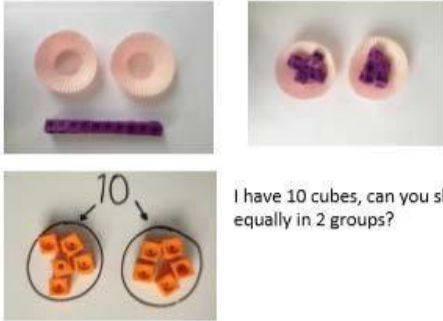
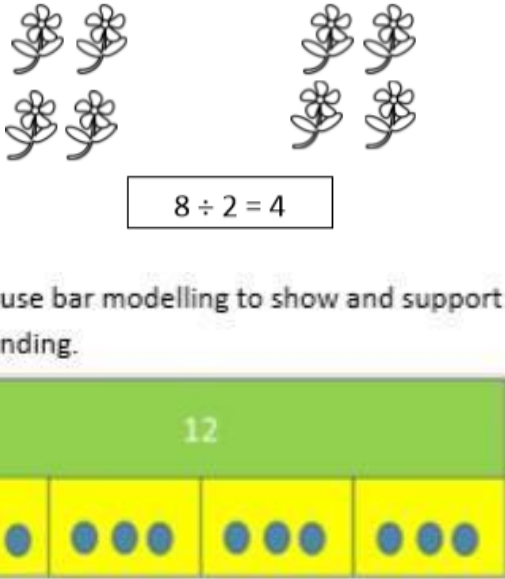
Bar model with pictures, symbols or icons to support understanding of finding 2 equal parts of a number, to show how two halves make a whole.



Learners create and visualise 3 or more equal groups WHEN children are ready.

Division - commonly seen in Year 1

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array

Concrete representations	Pictorial representations	Abstract representations
 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding. $12 \div 4 = 3$	Share 9 buns between three people. $9 \div 3 = 3$

Division - commonly seen in Year 2

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

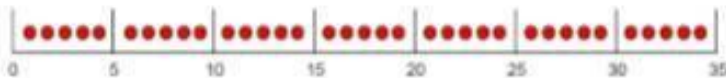
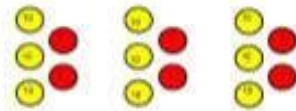
Concrete representations

Divide quantities into equal groups.

Use cubes, counters, objects or place value counters to aid understanding.

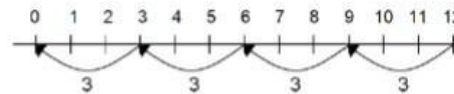


$$96 \div 3 = 32$$



Pictorial representations

Use a number line to show jumps in groups. The number of jumps equals the number of groups.



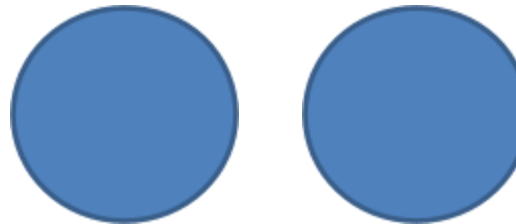
Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.



$$20 \div 5 = ?$$

$$5 \times ? = 20$$

$$12 \div 2$$



Cross-off the pictures and place the result in the circle.

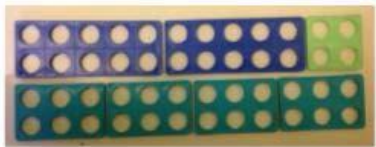
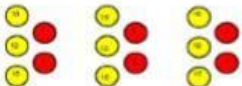
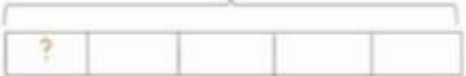
Abstract representations

$$28 \div 7 = 4$$

Divide 28 into 7 groups. How many are in each group?

Division - commonly seen in Year 3

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product

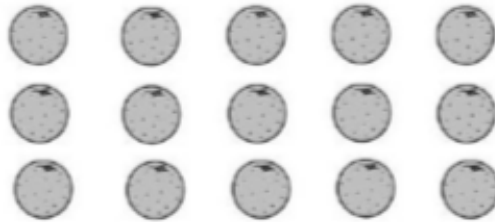
Concrete representations	Pictorial representations	Abstract representations
Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 $96 \div 3 = 32$ 	Continue to use bar modelling to aid solving division problems.  $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$

Draw an array and use lines to split the array into groups to make multiplication and division sentences



Find the inverse of multiplication and division sentences by creating eight linking number sentences.

$$7 \times 4 = 28$$

$$4 \times 7 = 28$$

$$28 \div 7 = 4$$

$$28 \div 4 = 7$$

$$28 = 7 \times 4$$

$$28 = 4 \times 7$$

$$4 = 28 \div 7$$

$$7 = 28 \div 4$$

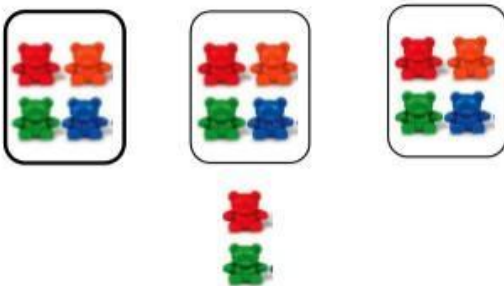
Tens		Ones	
10	10	1	1
10	10	1	1
10	10	1	1

He shares the tens first and exchanges the remaining ten for ones.

Then he shares the ones.

$42 \div 3 = 14$

Divide objects between groups and see how much is left over



Use place value counters to work out $94 \div 4$
Did you need to exchange any tens for ones?
Is there a remainder?



Tens	Ones

Tens	Ones
10	1 1 1 1
10	1 1 1 1
10	1 1 1 1

$42 \div 3$

$30 \div 3$

$12 \div 3$

 $48 \div 4 \bigcirc 36 \div 3$

$52 \div 4 \bigcirc 42 \div 3$

$60 \div 3 \bigcirc 60 \div 4$

$29 \div 8 = 3 \text{ REMAINDER } 5$
 ↑ ↑ ↑ ↑
 dividend divisor quotient remainder

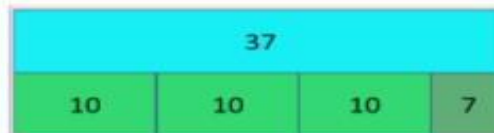
Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



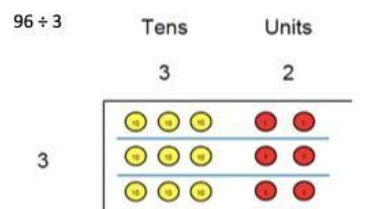
Use bar models to show division with remainders.



Division - commonly seen in Year 4

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive

Concrete representations



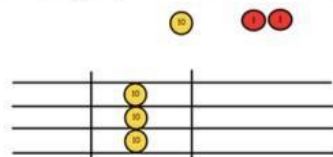
Use place value counters to divide using the bus stop method alongside



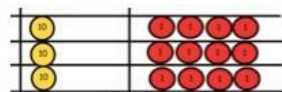
Calculations
 $42 \div 3$

$42 \div 3 =$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



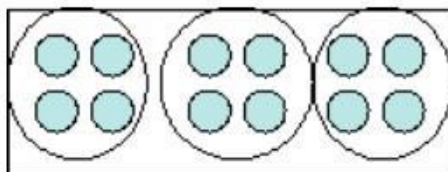
We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Pictorial representations

Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Abstract representations

Begin with divisions that divide equally with no remainder.

Long division (to illustrate chunking) then leading to short division. Long division examples can be seen on p42.

Short division.

$$\begin{array}{r} 19 \\ 4 \overline{) 76} \end{array}$$

$$\begin{array}{r} 247 \\ 3 \overline{) 741} \end{array}$$

Recognise that a 0 is used as a place holder, if the number is not divisible.

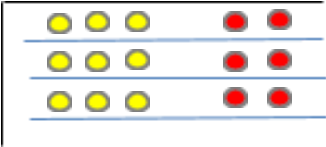
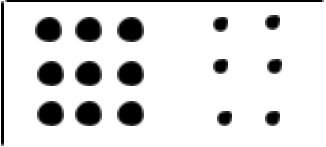
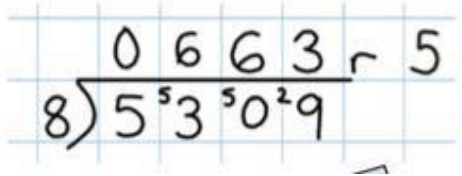
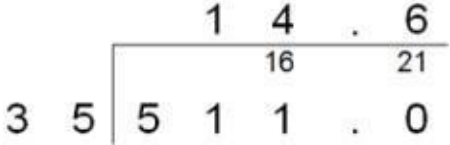
$$\begin{array}{r} 093 \\ 8 \overline{) 744} \end{array}$$

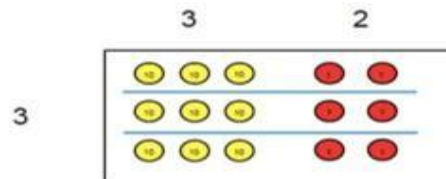
Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$$

Division - commonly seen in Year 5

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

Concrete representations	Pictorial representations	Abstract representations
$96 \div 3$ Tens Ones	Continue to use drawn diagrams with dots or circles to support division of numbers into equal groups. Move towards counting in multiples to divide more efficiently. $96 \div 3$ Tens Ones 3  90 6 3 	 Finally move into decimal places to divide the total accurately.  TEACH LONG DIVISION ALONGSIDE SHORT DIVISION – see page 42 for examples.

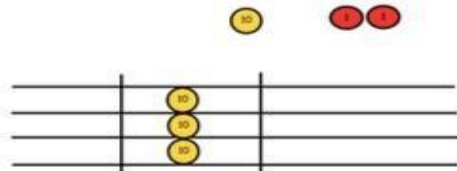


Use place value counters to divide using the bus stop method alongside

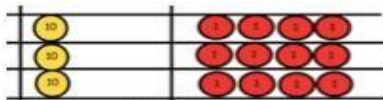


$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups.



We look how much in 1 group so the answer is 14.

Division - commonly seen in Year 6

Vocabulary share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

Abstract representations

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

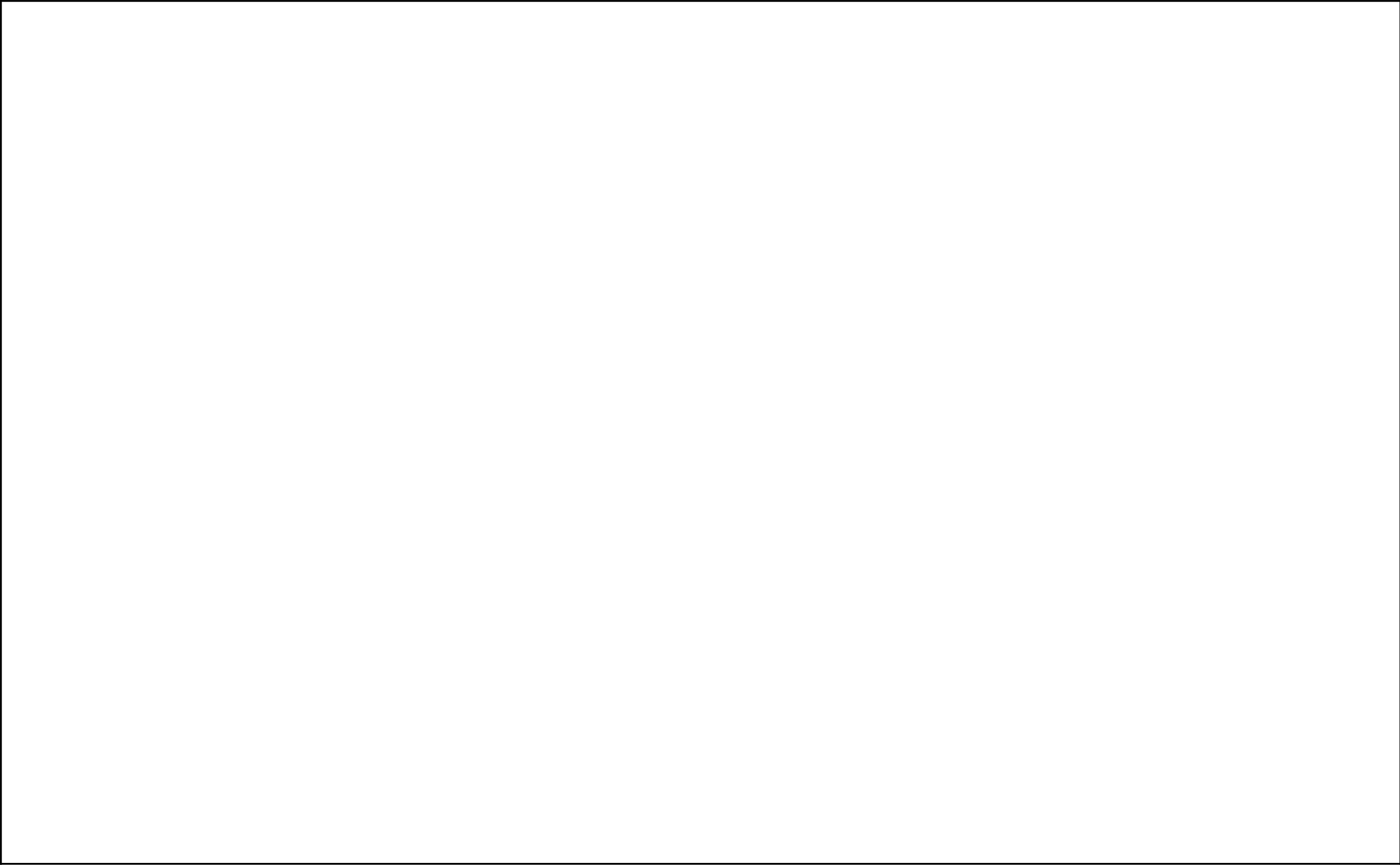
When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$



1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \end{array}$ Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 1 \end{array}$ To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \downarrow \\ \hline 18 \end{array}$ Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ The division is over since there are no more digits in the dividend. The quotient is 29.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$ Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ -2 \\ \hline 0 \end{array}$ Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ Next, drop down the 7 of the tens next to the zero.
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \end{array}$ Divide 2 into 7. Place 3 into the quotient.	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ Divide 2 into 18. Place 9 into the quotient.	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ There are no more digits to drop down. The quotient is 139.

