Clapham Manor Primary School & Nursery

Growing Brighter Futures

Maths intent/implementation/impact statement and Calculation Policy

Intent

The Mathematics National Curriculum (DfE, September 28th, 2021) states that, mathematics is a creative and highly interconnected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject. We subscribe to this belief so, mathematics forms an important part of our high-quality, broad and balanced curriculum where we endeavour to ensure that learners develop an enjoyment and enthusiasm for maths that will stay with them throughout their lives and their life-long learning.

To achieve this, we focus on ensuring that:

- Every adult believes that learners can succeed and seeks to remove or minimise , wherever possible, barriers to that success;
- our Maths curriculum provides breadth and balance, is relevant and engaging and is differentiated to match the needs and abilities of all our children to ensure that all pupils are able to excel;
- we are following the concrete-pictorial-abstract[1] approach as a means to developing a solid understanding of mathematical concepts throughout the school;
- learners understand rather than just do;
- learners become fluent;
- learners have the opportunity to apply their learning, reason and solve problems;
- learners to develop personal qualities such as resilience, independent thinking, cooperation and self-confidence in mathematics

Implementation

[1] Concrete representation – a hand- on, practical component using real objects, which is a foundation for conceptual understanding and essential whatever age or current mathematical progress. Pictorial representation – relating the hands-on experiences to representations, such as diagrams or pictures. Abstract representation – representation of calculations or problems using mathematical notation.

From EYFS to Year 6:

- we adhere to our calculation policy, which outlines the progression of strategies and methods to be taught.
- learning follows the 'White Rose' learning progression, although we have blended year groups to make a progressive curriculum fit for a small-school, which supports our learners in learning the fundamentals behind the meanings of numbers and exploring other key mathematical areas. We have also identified Key Learning Foci (KLF), which we believe are crucial for our learners. These KLF are incorporated into our blended curriculum.
- we adapt learning materials and activities to suit and benefit the needs of our learners.
- our maths curriculum is also supported with other suitable resources such as NRiCH, 'Classroom Secrets' and self-created resources;
- learning opportunities both inside and outside the classroom are planned for and encouraged throughout the year groups;
- where additional support or challenge is required, 'scaffolds' and deeper thinking challenges are used to support children further to ensure that they have secured the small step before moving on. These 'scaffolds' may, for example, be in the form of returning to concrete resources or pictorial representations, revisit previous learning to consolidate, pre-teaching, post-teaching, learning clubs or adult support;
- Adults, regularly, explicitly link current learning to other parts of the maths curriculum, previous learning, real-life application and other areas of the curriculum such as history.
- Adults use modelling to the learning and application of skills;
- AfL is used by adults during teaching sessions and learning is adapted accordingly;
- Learners have access, both at school and at home, to online learning such as Timestable Rockstars and are encouraged to engage with those.
- Termly assessments/progress checks (informal and/or formal)/ observations are used as a diagnostic tool to ensure that teachers are adapting learning to meet the needs of all learners and ensure that any necessary interventions are targeted specifically to meet the needs of children;
- Learners develop their fluency in rapid recall of times-tables and related division facts up to 12 x 12 by the end of year 6 as well as learning how to apply and manipulate their understanding of these to reason and solve problems. In addition to this, learners develop an understanding how other related facts can be found using their knowledge of times-tables and linked division facts, such as if they know 5 x 4 = 20 then 50 x 4 = 200 or 5 x 0.4 = 2.

Impact

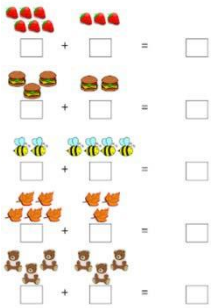
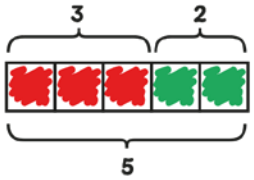
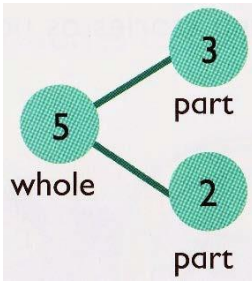
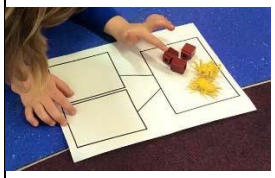
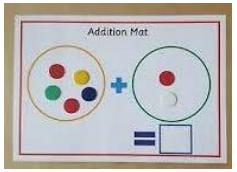
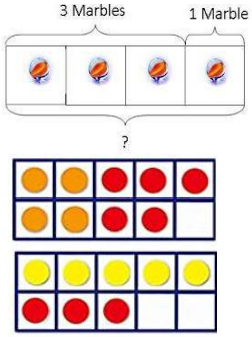
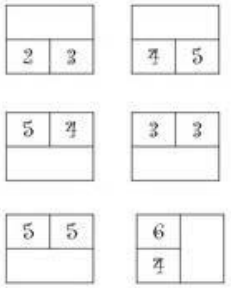
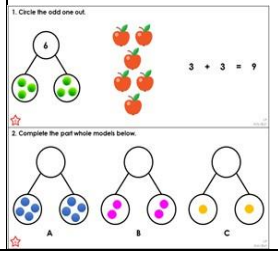
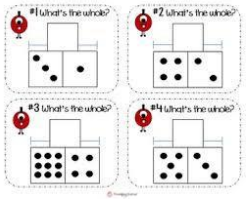
By the end of Year 6, transitioning to secondary school, we aspire that a Clapham Manor mathematician will have developed:

- an understanding of how their brain works and how this impacts on learning in mathematics;.
- a 'learning from mistakes' attitude towards learning;
- mathematical language to be able to talk about their understanding and learning;
- the confidence to tell an adult that learning is too tricky and use resources available to support their understanding;
- the confidence to tell an adult that learning is too simple, prove it and then welcome further challenge;
- a bank of efficient and accurate mental and written skills that can be used to calculate efficiently.
- the skills to be able to apply their learning in a range of problem-solving and reasoning situations with confidence and resilience;
- the language to be able to discuss, answer, prove and explain (A.P.E) their opinions and solutions.

Addition

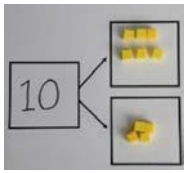
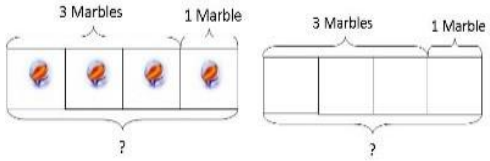
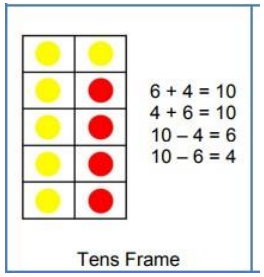
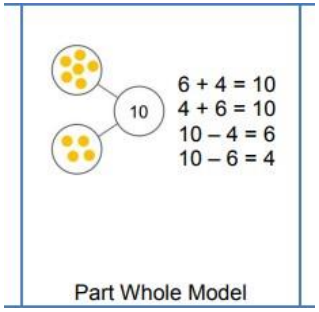
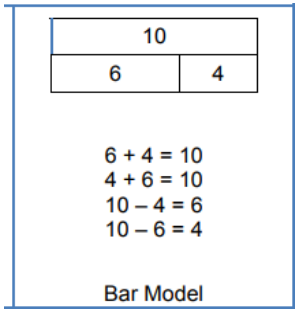
Addition - commonly seen in EYFS

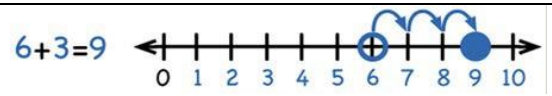
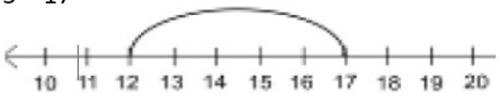
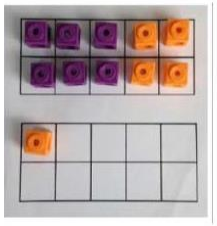
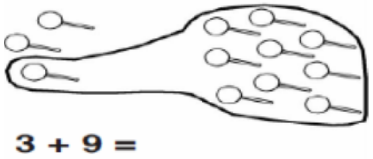
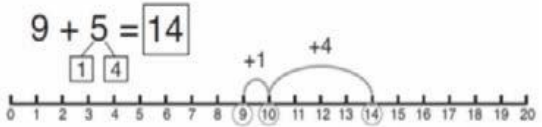
Vocabulary

Concrete representations	Pictorial representations	Abstract representations
   Use a range of different objects to physically manipulate, group and regroup.	  Pictures of two groups to enable counting to find the total.	Using symbols and numbers to form calculations. $5+2=7$
 Use maths resources such as Numicon, multilink, counters etc to support understanding.	 Bar model using visuals, pictures, symbols or colours.	
  Use visual supports such as 10 frames, part-part-whole diagrams, addition mats with objects.	 Use visuals such as ten frames, part-part-whole andaddition mats with pictures /symbols.	
	 	* No expectation that number sentences/ calculations are recorded.

Addition - commonly seen in Year 1

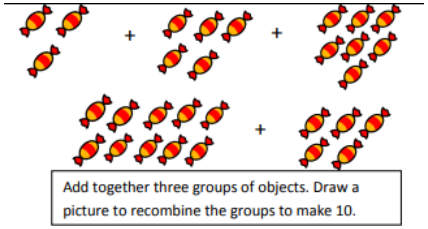
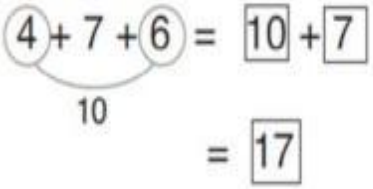
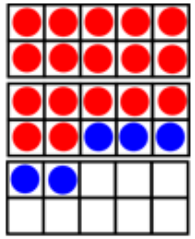
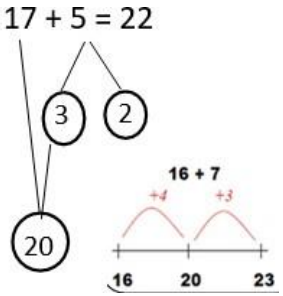
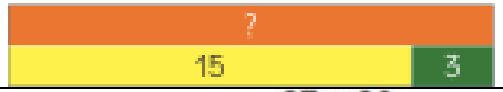
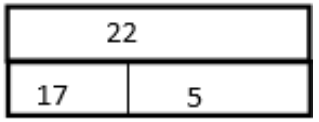
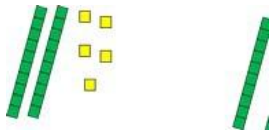
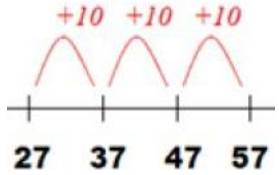
Vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole

Concrete representations	Pictorial representations	Abstract representations
 Use cubes, Numicon or other objects to add two numbers together as a group or in a bar.  Use part-part-whole model	 Use pictures to add two numbers together as a group or in a bar.  Part-part-whole and Bar Models used to support problem solving involving addition, with concrete and pictorial representations being used alongside each other.  Pictorial (concrete) Abstract	 Use the part-part whole diagram as shown above to move into the abstract. **4 + 3 = 7** **10= 6 + 4**
 Use counters, Numicon or other objects, e.g. counters in egg boxes to support understanding.		 Bar model and part-part-whole used alongside abstract.

 Start with the higher number on the beadstring/Numicon/tens frame etc and then add the lower number by counting on (1 by 1) to find the answer.	 Start at the higher number on the number line and count on in ones. 12 + 5 = 17 	5 + 12 = 17 17= 12 + 5 Place the higher number in your head and add on the lower number to find your answer.
 6 + 5 = 11 Starting with the higher number, use the smaller number to count on to make 10. 	 3 + 9 = Use pictures or a number line. Regroup or Partition the lower number using the part-part-whole model to make 10.  Starting at the higher number on the number line, count on in ones, in jumps to the next 'friendly; (multiple of 10) number and beyond or in one jump.	7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?

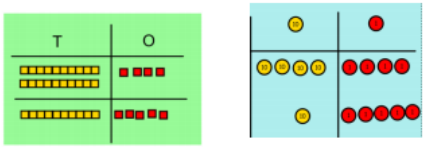
Addition - commonly seen in Year 2

Vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

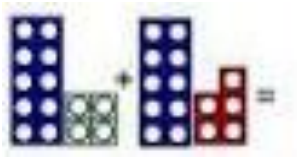
Concrete representations	Pictorial representations	Abstract representations
4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (where possible) then add on the 3rd digit.	 	 Combine the two numbers that make 10 and then add on the remainder.
 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model.  Bar Model  	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
 25 + 10 = 35 Explore how the ones digit/value does not change.	**27 + 30**  Dienes or other resources may be used alongside the number line. Calculation shown alongside the number line to make links.	27 + 10 = 37 27 + 20 = 47 27 + □ = 57

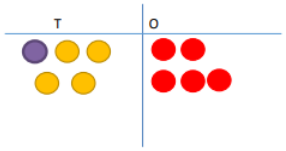
24 + 15=

Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.

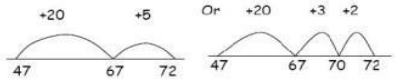


Numicon may also be used by learners if needed.





Moving away from practical resources to symbols such as drawn counters to help them to solve calculations.



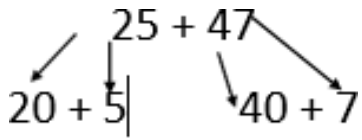
Use number line and bridge ten using partwhole if necessary. Numicon, Dienes etc may be used above the number line if needed.

The calculation shown alongside the number line to see the connection.

Model	Calculation

<u>The Bar Model</u> used to support problem solving moving onto the generalisation that b+c = a. Abstract representation alongside the pictorial to support where necessary.

Partitioning:



20 + 40 = 60

5+ 7 =12

60 + 12 = 72

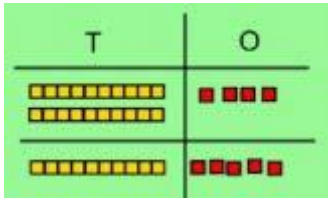
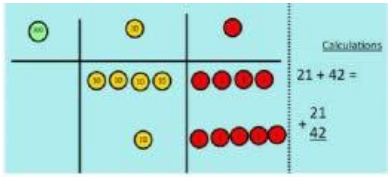
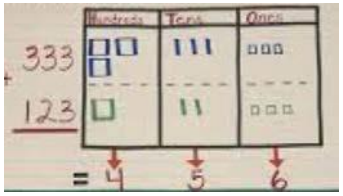
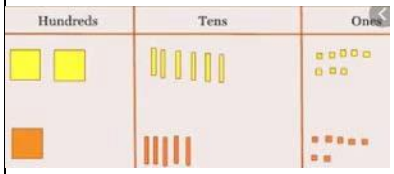
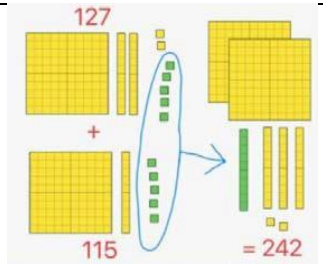
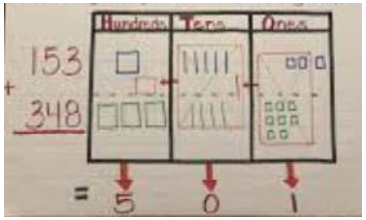
Recording addition in columns supportsplace value and prepares for formal written methods with larger numbers.

Toward the end of the year, learners move towards more formal recording using partitioning method:

$$\begin{array}{r} 40 + 7 \\ 30 + 5 \\ \hline 70 + 12 \end{array}$$

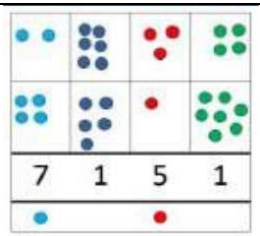
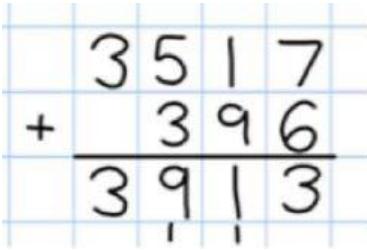
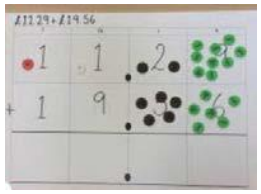
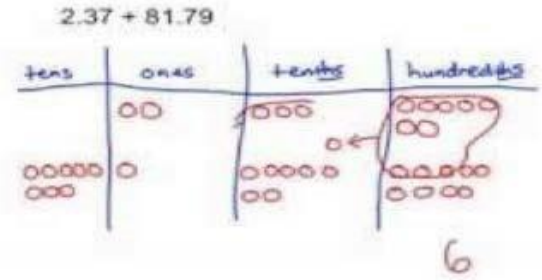
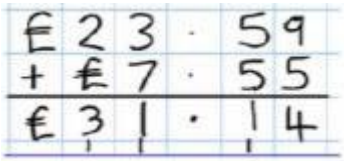
Addition - commonly seen in Year 3

Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Concrete representations	Pictorial representations	Abstract representations
 Using manipulatives (Dienes, Numicon, counters etc), learners line up hundreds, tens and ones.  Learners should be secure with using PV countersbefore moving onto pictorial. The calculation shown alongside the model used to see the connection.	 Learners draw in a PV frame, the manipulatives they are using. Secure knowledge of representation withthe PV columns. The calculation will be shown alongside themodel to see the connection	2 2 3 + 1 1 4 3 3 7 Learners to move towards recording moreformally. Some learners may need to use the expanded method (see below).
  Exchange ten ones for a ten. Model using Dienes,Numicon and place value counters.	 Children can draw a representation of thegrid to further support their understanding, carrying the ten ***underneath*** the line.	20 + 5 40 + 8 60 + 13 = 73 Learners begin with the abstract expanded form. When confident, learners move on to the formal method. 536 + 85 621 11

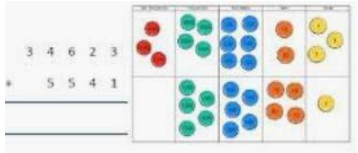
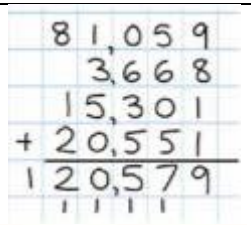
Addition - commonly seen in Year 4

Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary

Concrete representations	Pictorial representations	Abstract representations
 Learners continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. The calculation shown alongside the manipulative to show the links.	 Learners draw a pictorial representation of the columns and place value counters to make links.	 Continue from previous work to carryhundreds as well as tens.
 Introduce decmial place value counters and model exchange for addition.		 As learners move on, introduce decimals with the same number of decimal places and different. Money used here.

Addition - commonly seen in Year 5/6

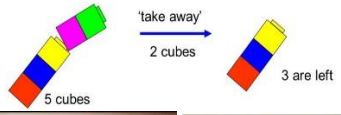
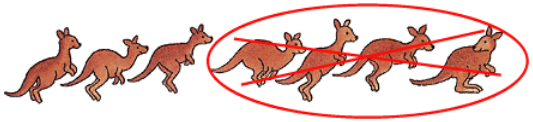
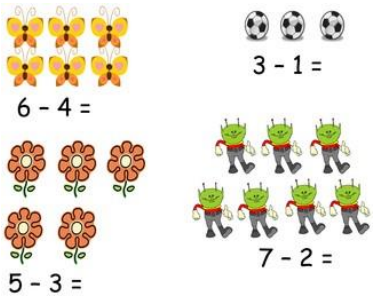
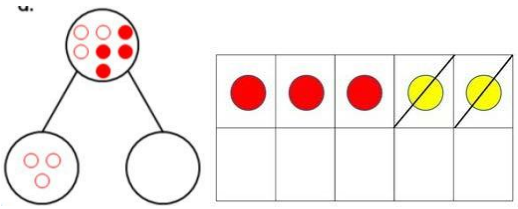
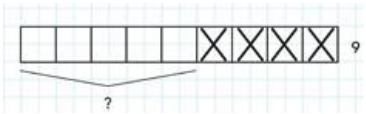
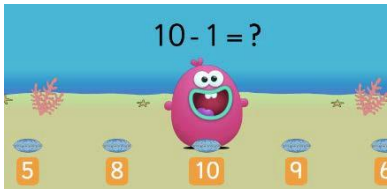
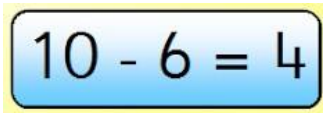
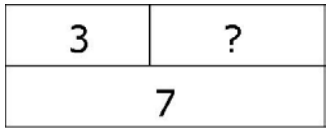
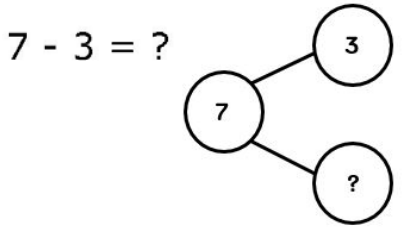
Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens boundary, hundreds boundary, decimal,decimal point

Concrete representations	Pictorial representations	Abstract representations
See Year 4	See Year 4	 Abstract supported by a pictorial or concrete if needed.
See Year 4	See Year 4	 Use zeros for place holders.

Subtraction

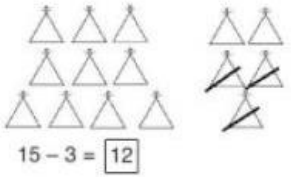
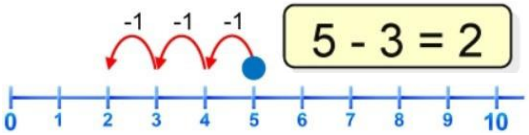
Subtraction - commonly seen in EYFS

Vocabulary subtract, take away, equal, less, fewer, minus, leaves.

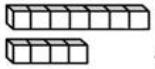
Concrete representations	Pictorial representations	Abstract representations
   Use toys and other objects forchildren to physically manipulate, group/regroup.    Use specific maths resources such as unifix, Numicon, bead strings etc.   Use visual supports such as ten frames, part-part-whole and subtraction mats, with the objects and resources that can be manipulated.	  A group of pictures for children to cross out or cover quantities to support subtraction.   Use visual supports such as ten frames, part-part-whole and bar model with pictures/icons.	A focus on symbols and numbers to form a calculation.     * No expectation that number sentences/ calculations are recorded.

Subtraction- commonly seen in Year 1

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, howmany left, how much less is…

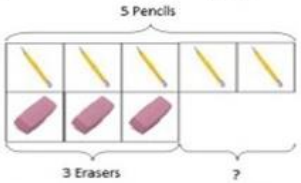
Concrete representations	Pictorial representations	Abstract representations
Use physical objects, counters , cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2 Start with the higher number on the beadstring/ Numicon/ counters/ tens frame etc and then subtract the lower number by counting onback (1 by 1) to find the answer. **13 – 4**  Use counters and move them away from the group as counting backwards.  COUNT ON INSTEAD???	  Cross out drawn objects to illustrate what has been taken away.  Count back on a number line or track Start at the higher number and count back the lower number showing the jumps on the number line.	7—4 = 3 16—9 = 7 Put 12 in your head, count back 3. What number are you at?

Compare objects and amounts

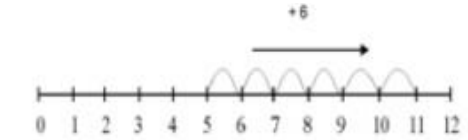


'Seven is 3 more than four'

'I am 2 years older than my sister'



Lay objects to represent bar model.

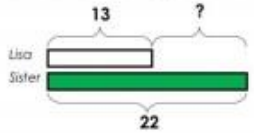


Count on to find the difference.

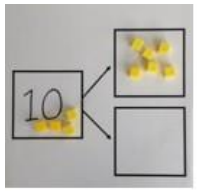
Comparison Bar Models

Draw bars to find the difference between 2 numbers.

Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.



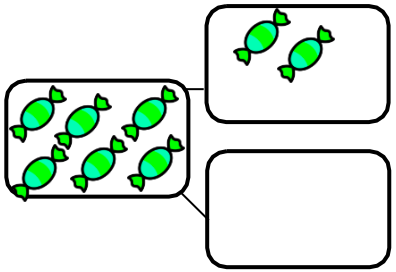
Harry has 14 sweets and her brother has 8.
How many more does Harry have than her brother?



Link to addition. Use PPW model to model the inverse.

If 10 is the whole and 6 is one of the arts, what s the other part?

10—6 = 4

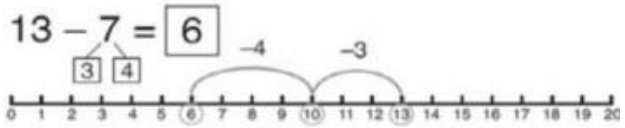


Use a pictorial representation of objects to show the part-part-whole model

Move to using numbers within the part whole model.

14 – 9 =

Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.



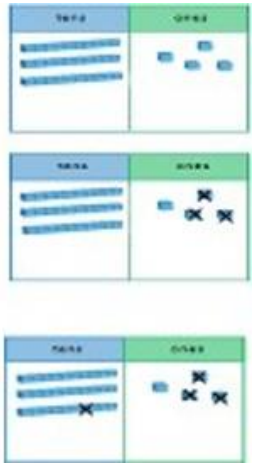
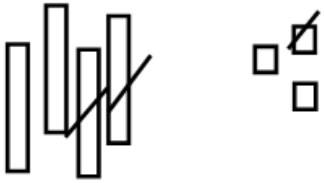
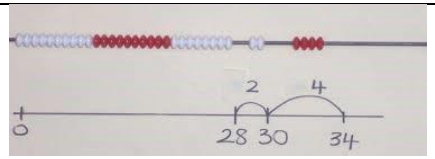
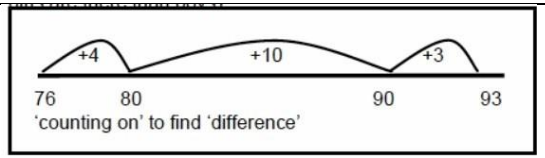
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.

16 – 9=

How many do we take off to reach thenext 10?
How many do we have left to take off?

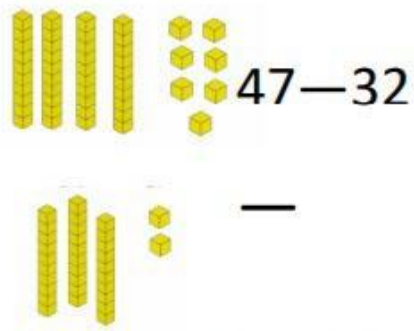
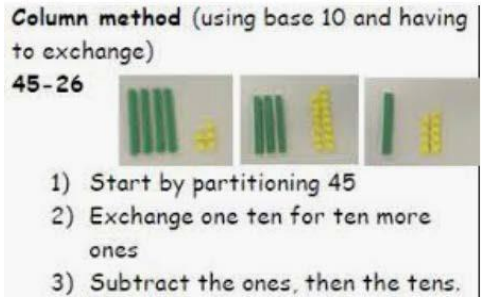
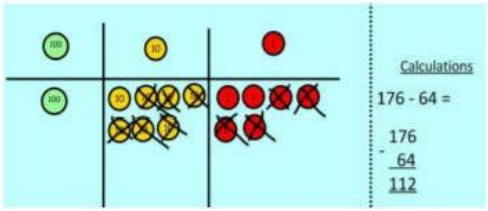
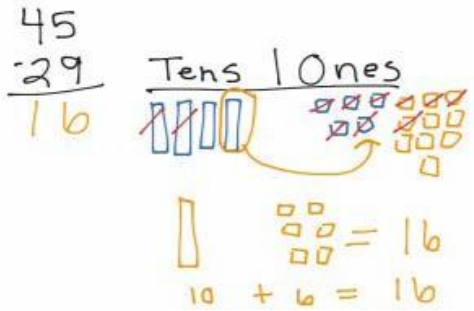
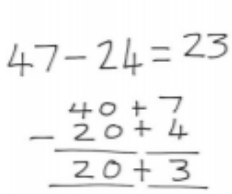
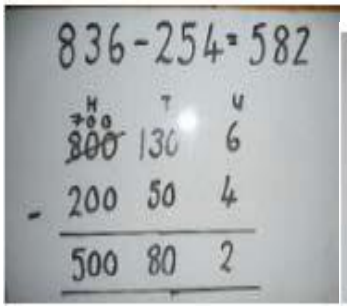
Subtraction – commonly seen in Year 2

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units

Concrete representations	Pictorial representations	Abstract representations
34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping. Calculation shown alongside the manipulative used.	 43—21 = 22 Learners draw representations of Dienes and cross off.	43—21 = 22 *Recording subtraction in columns supporting place value and preparing for formal written methods with higher numbers.* Toward the end of the year, learners move to more formal recording using partitioning method: e.g. 76-24=52 70 and 6 -20 and 4 50 and 2
 34—28 Use a bead bar or bead strings to model counting to the next ten and the rest.	 Use a number line to count on to next ten andthen the rest.	73—56 = 17

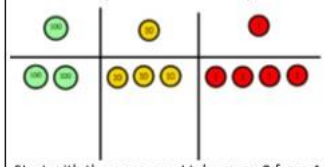
Subtraction – commonly seen in Year 3

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how much less is…difference, count on, strategy, partition, tens units

Concrete representations	Pictorial representations	Abstract representations
 Use base 10 or Numicon to model The calculation will be shown alongside themodel chosen to make links. Begin with Dienes or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange. 	Children are to be secure with use of PVcounters before moving onto abstract.   When confident, learners can find their own way to record the exchange/regrouping	Begin with the expanded form then moving on to the formal method.    

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

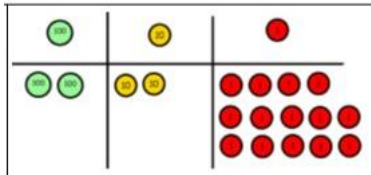
Make the larger number with the place value counters



Calculations

234
- 88

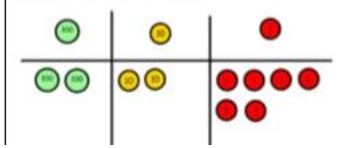
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

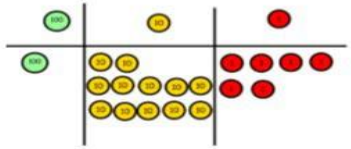
Now I can subtract my ones.



Calculations

234
- 88

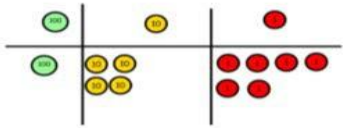
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Calculations

234
- 88

Now I can take away eight tens and complete my subtraction



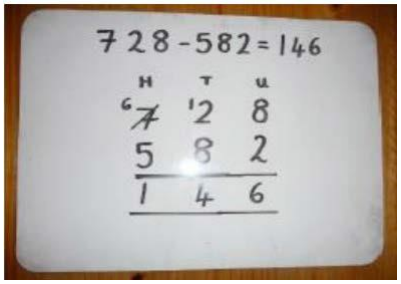
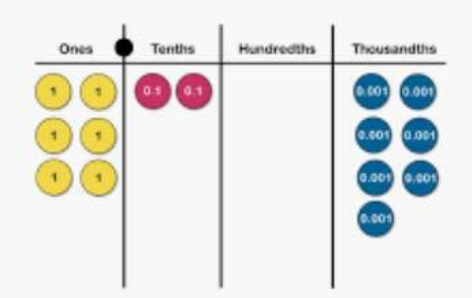
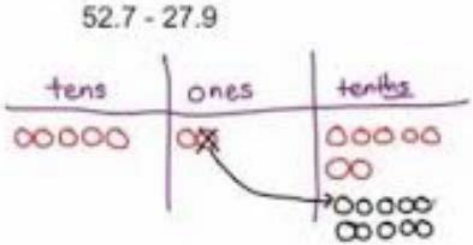
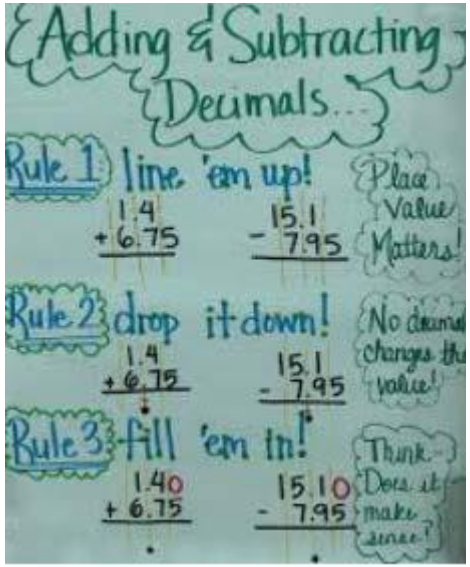
Calculations

234
- 88
146

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

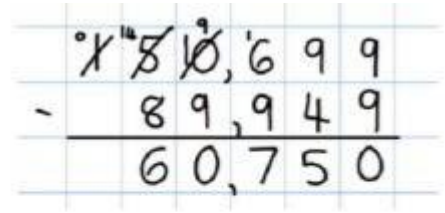
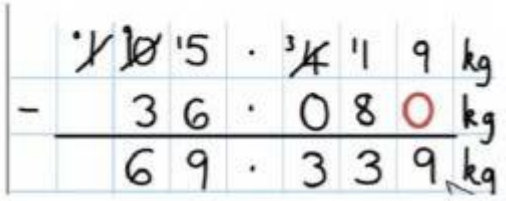
Subtraction – commonly seen in Year 4

Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units

Concrete representations	Pictorial representations	Abstract representations
Model process of exchange using Numicon, base ten and then move to using place value counters. Use the phrase 'take and make' for exchange - see Y3 The calculation will be shown alongside the model chosen to show the links.	Learners to draw place value counters and show their exchange — see Y3 The calculation will be shown alongside the model chosen to show the links.	 This will lead to an understanding of subtractingany number including decimals.
Learners to be encouraged to use counters to represent numbers andtake counters away to subtract. 	 When confident, learners can find their own way to record the exchange/regrouping.	

Subtraction – commonly seen in Year 5/6

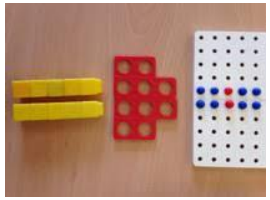
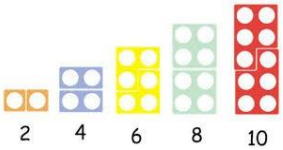
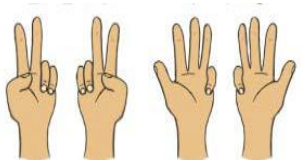
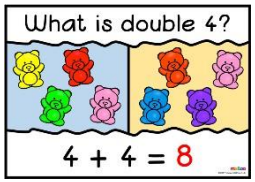
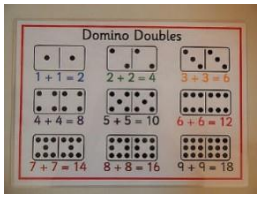
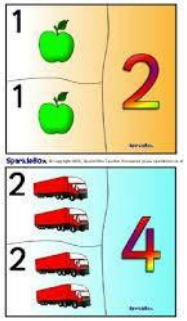
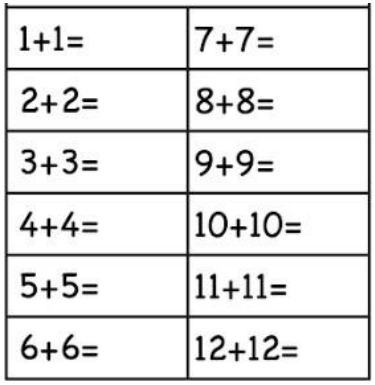
Vocabulary equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is…difference, count on, strategy, partition, tens units

Concrete representations	Pictorial representations	Abstract representations
See Year 4	See Year 4	 

Multiplication

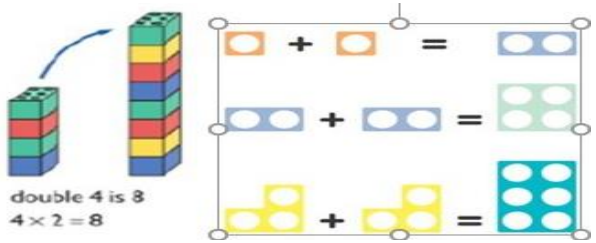
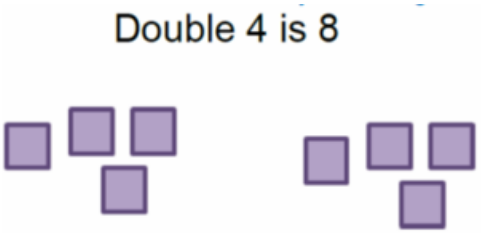
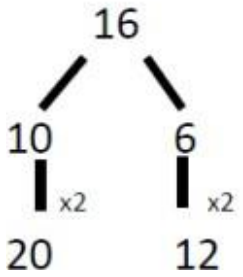
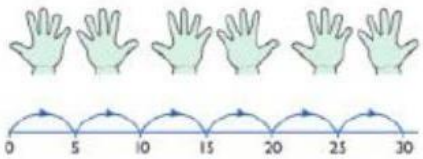
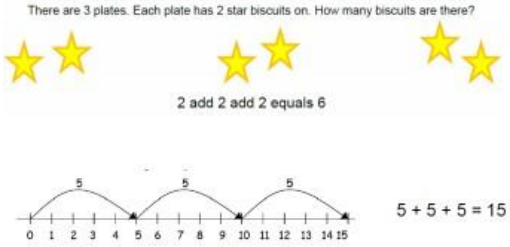
Multiplication - commonly seen in EYFS

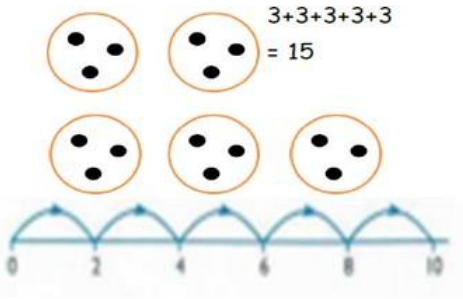
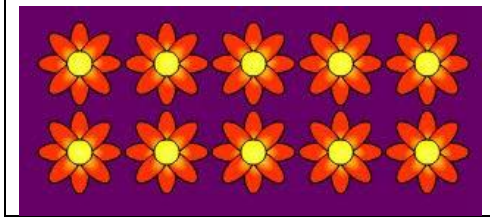
Vocabulary

Concrete representations	Pictorial representations	Abstract representations
  Using a variety of maths resources and objects, learners make 2 equal groups.     Learners explore physical and real-life examples to explore the concept of doubling as adding two equal groups.	   Use pictures, icons and symbols that encourage learners to explore the concept of doubling as adding two equal groups.	 Addition calculations to model adding two equal groups.

Multiplication – commonly seen in Year 1

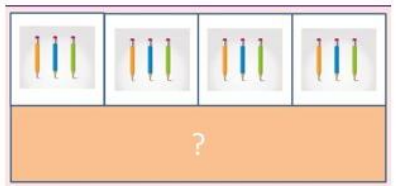
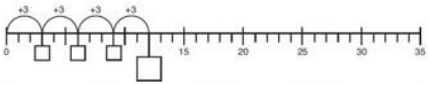
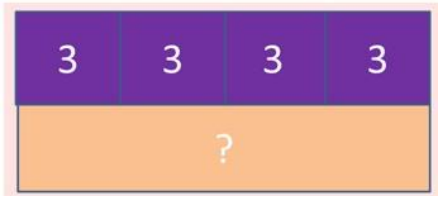
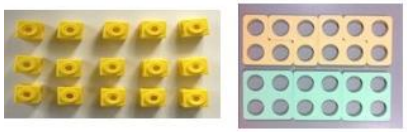
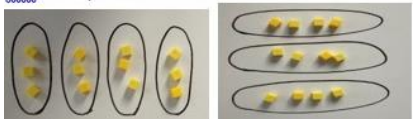
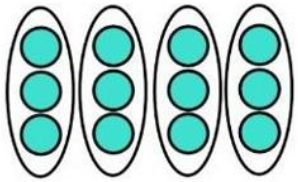
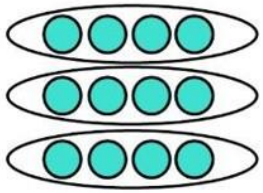
Vocabulary Groups of, lots of, times, array, altogether, multiply, double, equal

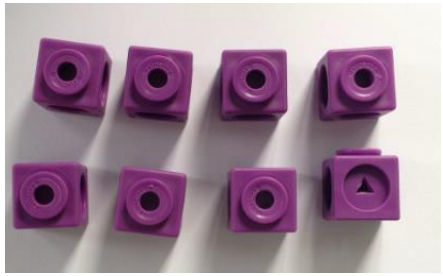
Concrete	Pictorial	Abstract
 Learners use practical resources, e.g. cubes and Numicon, to demonstrate doubling.	 Learns draw pictures to illustrate doubling.	Partition a number and then double each part before recombining it back together. 
 Calculate multiples using concrete objects and maths resources.	 Calculate multiples using a number line.	Count in multiples of a number aloud. Write sequences with multiples of numbers. **2, 4, 6, 8, 10** **5, 10, 15, 20, 25, 30**
 Use different objects to add equal groups.		Write addition sentences to describe objects and pictures. 

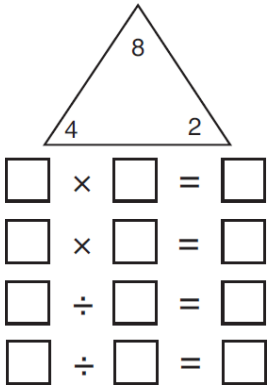
	Use pictures, icons or symbols including number lines to solve problems. There are 3 sweets in one bag. How many sweets in 5 bags?  	
Use objects laid out in arrays to find theanswers to 2 lots 5, 3 lots of 2 etc. 		3 x 2 = 6 2 x 5 = 10

Multiplication- commonly seen in Year 2

Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative.

Concrete representations	Pictorial representations	Abstract representations
Count the groups during skip counting, Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, bar models, counting sticks etc used to represent counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$
Commutativity Create arrays using counters, cubes, Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15





2 x 4 = 8

4 x 2 = 8

8 ÷ 2 = 4

8 ÷ 4 = 2

8 = 2 x 4

8 = 4 x 2

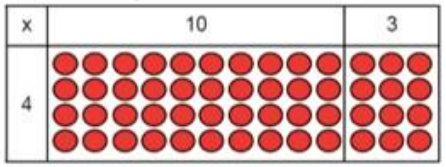
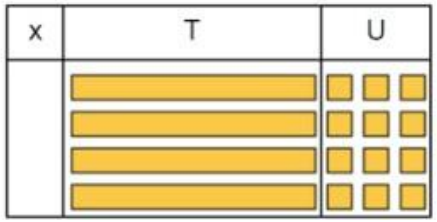
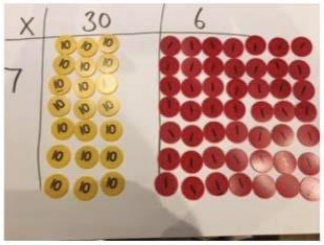
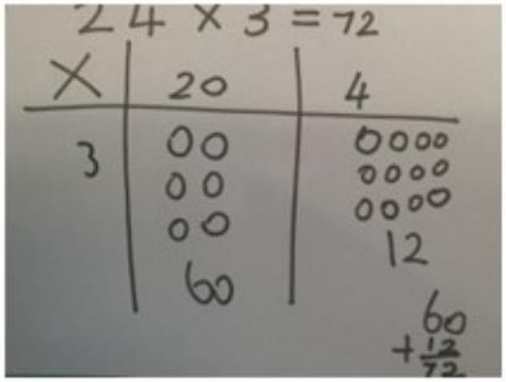
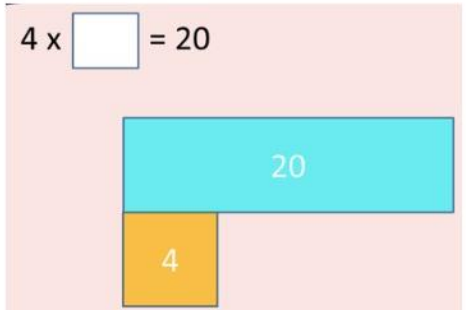
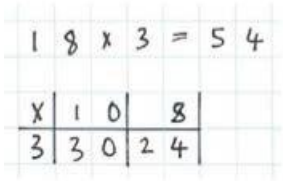
2 = 8 ÷ 4

4 = 8÷ 2

Show all 8 related fact family sentences.

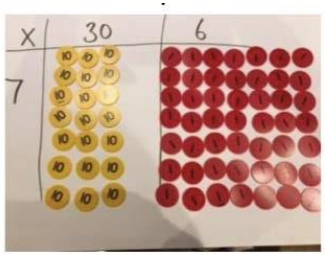
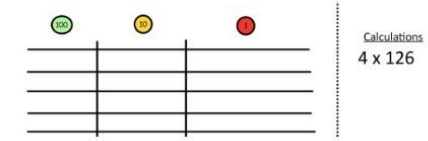
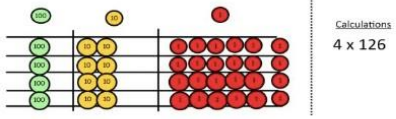
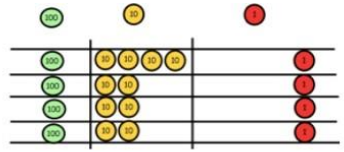
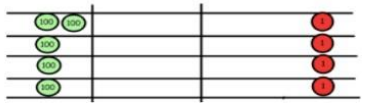
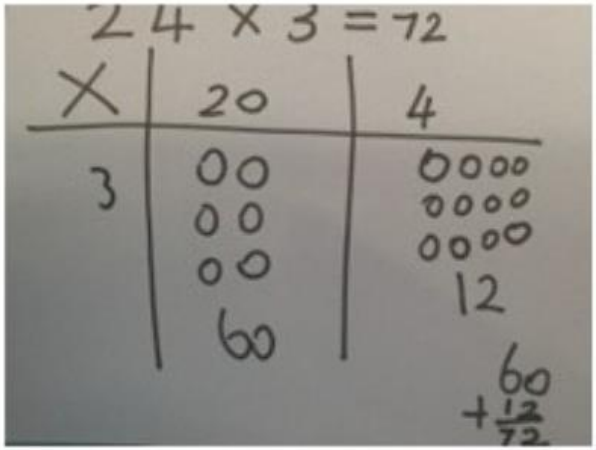
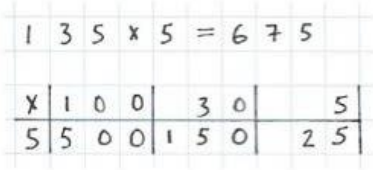
Multiplication - commonly seen in Year 3

Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up

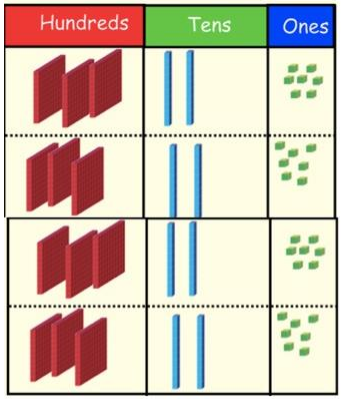
Concrete representations	Pictorial representations	Abstract representations
Show the link with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to using Base 10 to move towards a more compact method.  4 rows of 13 Move on to place value counters to showhow we are finding groups of a number.  Add up each column, starting with the ones making any exchanges needed. The calculation will be shown alongside the model chosen to show the links.	Learners can represent their work with place value counters and draw the colours (or just use circles) to show different amounts to show their thinking.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. TO x O  Children to add up each column to find the answer.

Multiplication - commonly seen in Year 4

Vocabulary Groups of, lots of, times, array, altogether, multiply, Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive

Concrete representations	Pictorial representations	Abstract representations
 Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.  Then you have your answer. 	Learners can represent their jottings with place value counters in a way that they understand. Can draw coloured or plain counters in different columns to show thinking. 	HTO x O  Learners to add up each column to find the answer.

Learners continue to be supported by place value counters at this stage of multiplication. Initially, this is done with no regrouping.

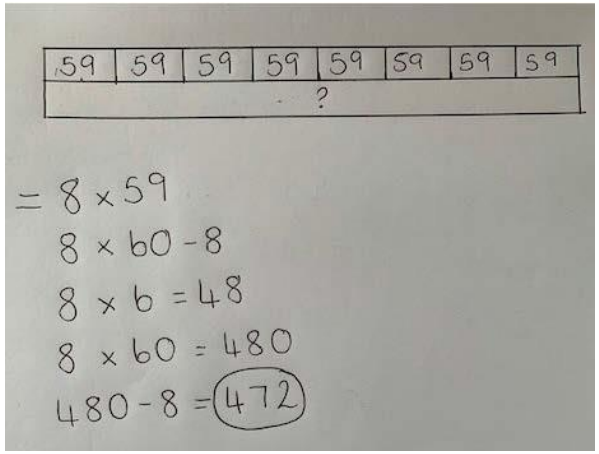


It is important at this stage that they always multiply the ones first.

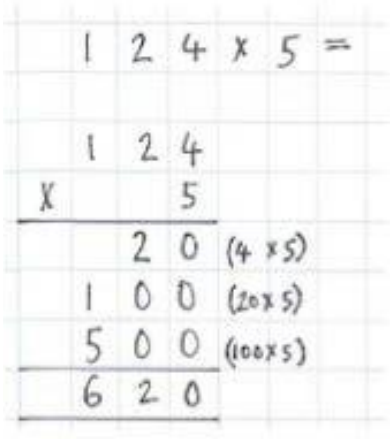
The corresponding long multiplication is modelled alongside

This grid method can be used to show how this relates to a formal written method.

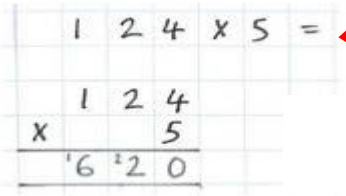
×	100	20	4
5	500	100	20



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

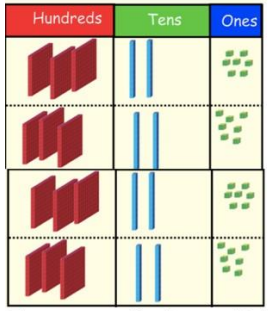
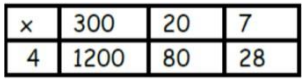
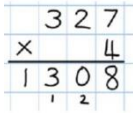
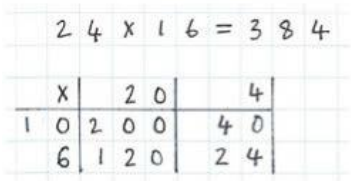
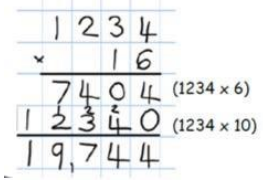


This may lead to a compact method.



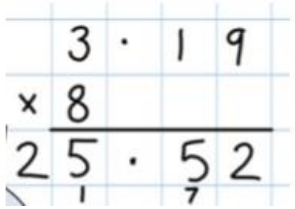
Multiplication – commonly seen in Year 5

Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed.

Concrete representations	Pictorial representations	Abstract representations
Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside		327 x 4 28 80 1200 1308  This may lead to a compact method.
Manipulatives used with the corresponding long multiplication modelled alongside.	Moving forward, multiply by a 2 digit number showing the different rows within the grid method. 	 24 x 6 on the first row. (6 x 4 = 24, carrying the 2 for the 20, then 6 x 2) 24 x 10 on the secondrow. Show multiplyingby 10 by putting zero in the units first. 

Multiplication- commonly seen in Year 6

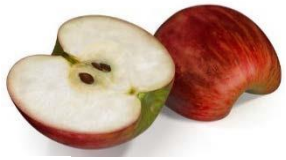
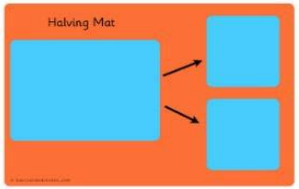
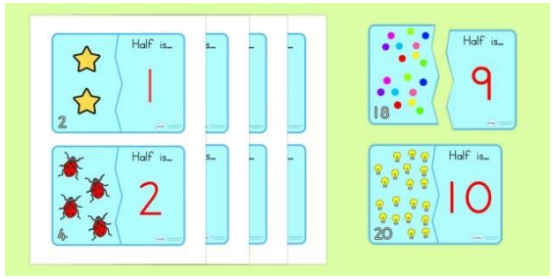
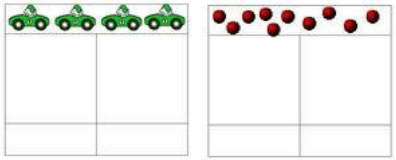
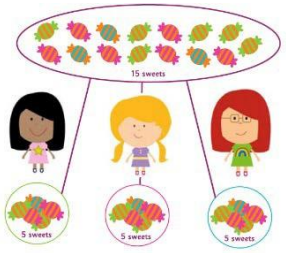
Vocabulary Groups of, lots of, times, array, altogether, multiply, multiplied by, repeated addition, sets of, equal groups, times as big as, commutative, product, multiples of, scale up, inverse, derive, factor pairs, composite numbers, prime number, factors, squared, cubed.

Concrete representations	Pictorial representations	Abstract representations
		Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Division

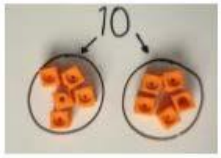
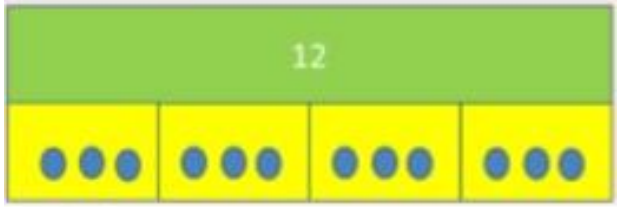
Division - commonly seen in EYFS

Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array, half, same

Concrete representations	Pictorial representations	Abstract representations
   Cut objects, food or shapes in half.  Counting and other maths resources for children to share intotwo equal groups.  Use visual supports such as halving mats and part-part-whole, with the physical objects and resources that can be manipulated.   Counting and other maths resources for children to explore sharing between 3 ormore.	 Use pictures, symbols and icons that show concept of halving in relation to subitising, addition and subtraction knowledge. E.g. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.  Bar model with pictures, symbols or icons to support understanding of finding 2 equal parts of a number, to show how two halves make a whole.  Learners create and visualise 3 or more equal groups WHEN children are ready.	

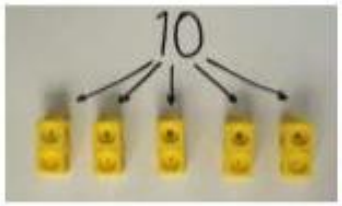
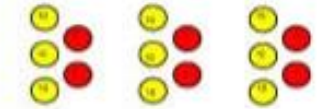
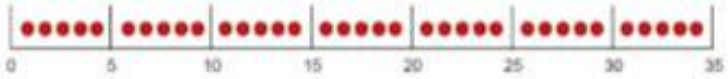
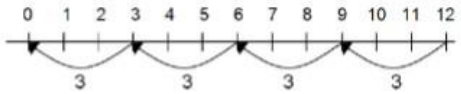
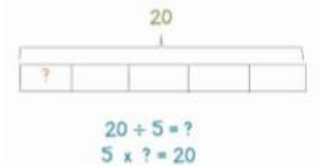
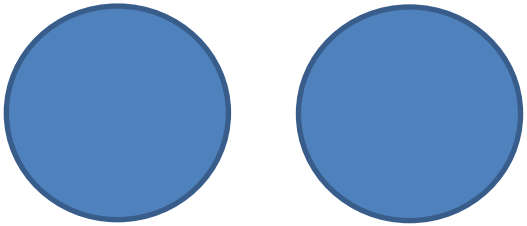
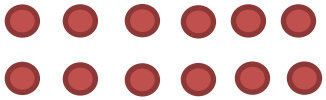
Division - commonly seen in Year 1

Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array

Concrete representations	Pictorial representations	Abstract representations
   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4 Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	**Share 9 buns between three people.** **9 ÷ 3 = 3**

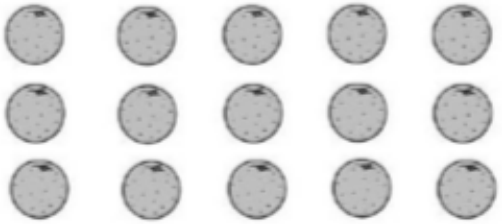
Division - commonly seen in Year 2

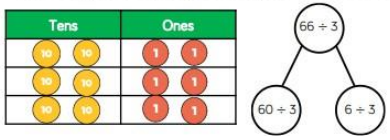
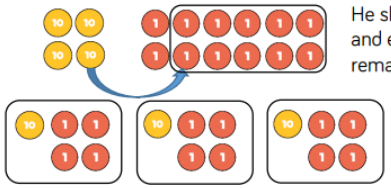
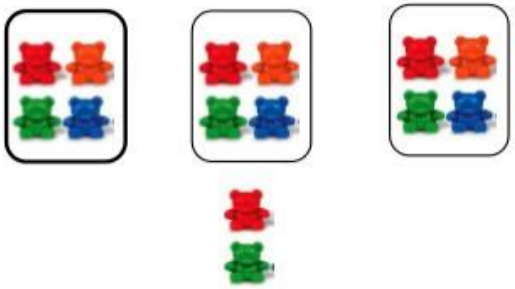
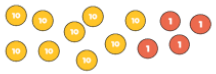
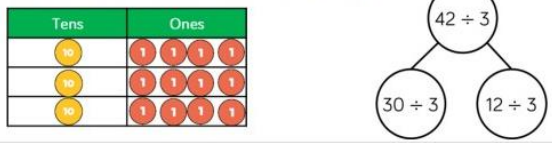
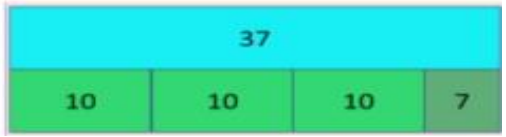
Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Concrete representations	Pictorial representations	Abstract representations
Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  96 ÷ 3 = 32   	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you ar dividing by and work out how many would be within each group.  12 - 2   Cross-off the pictures and place in the circle.	**28 ÷ 7 = 4** **Divide 28 into 7 groups. How many are in each group?**

Division - commonly seen in Year 3

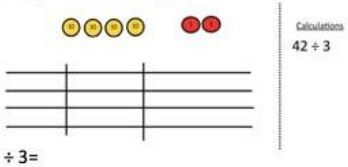
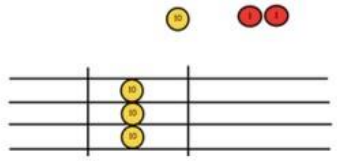
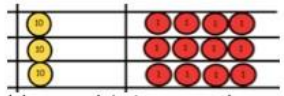
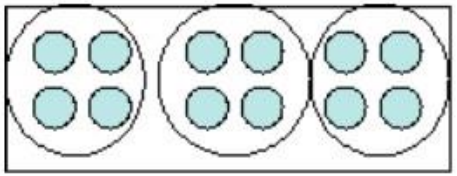
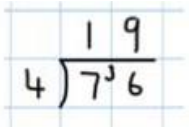
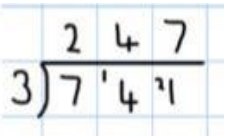
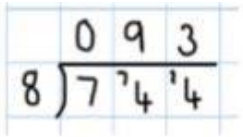
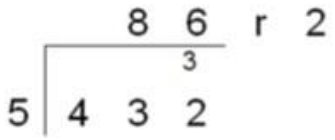
Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product

Concrete representations	Pictorial representations	Abstract representations
Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems.  20 ÷ 5 = ? 5 x ? = 20	How many groups of 6 in 24? 24 ÷ 6 = 4
 Link division to multiplication by creating an array and thinking about the number sentenc-es that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Eva uses a place value grid and part-whole model to solve 66 ÷ 3 	See part-part-whole model	
Ron uses place value counters to divide 42 into three equal groups.  He shares the tens first and exchanges the remaining ten for ones. Then he shares the ones. 42 ÷ 3 = 14 **14 ÷ 3 =** Divide objects between groups and see how much is left over  Moving on to: Use place value counters to work out 94 ÷ 4 Did you need to exchange any tens for ones? Is there a remainder?  Tens / Ones	Annie uses a similar method to divide 42 by 3  Use pictorial representation for the place value counters, alongside the part-whole model Learners use their times-tables to partition the number into multiples of the divisor. Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	96 ÷ 8 96 ÷ 4 96 ÷ 3 96 ÷ 6 Compare the statements using <, > or = 48 ÷ 4 ◯ 36 ÷ 3 52 ÷ 4 ◯ 42 ÷ 3 60 ÷ 3 ◯ 60 ÷ 4 Complete written divisions and show the remainder using r. 29 ÷ 8 = 3 REMAINDER 5 ↑ dividend ↑ divisor ↑ quotient ↑ remainder

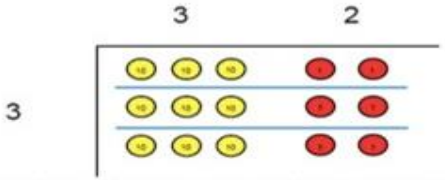
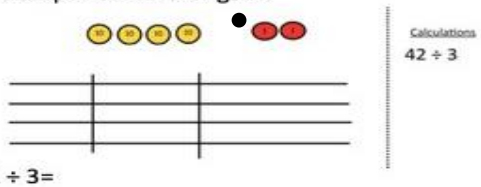
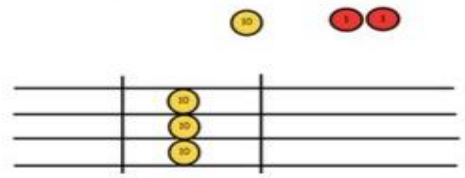
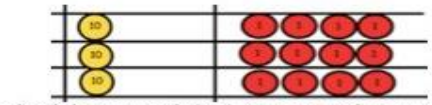
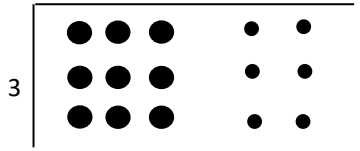
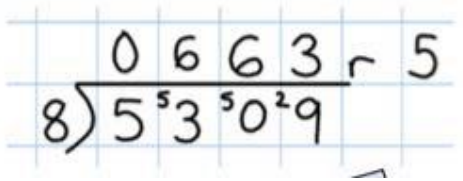
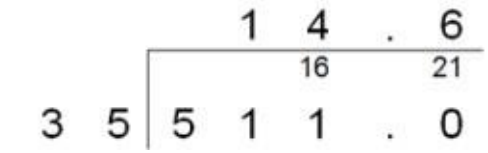
Division - commonly seen in Year 4

Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive

Concrete representations	Pictorial representations	Abstract representations
96 ÷ 3 Tens Units 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. Long division (to illustrate chunking) then leading to short division. Long division examples can be seen on p42. **Short division.**   Recognise that a 0 is used as a place holder, if the number is not divisible.  Move onto divisions with a remainder. 

Division - commonly seen in Year 5

Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts, inverse, derive, formal written method.

Concrete representations	Pictorial representations	Abstract representations
96 ÷ 3 Tens Ones 3 2 3  Use place value counters to divide using the bus stop method alongside  Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Continue to use drawn diagrams with dots or circles to support division of numbers into equal groups. Move towards counting in multiples to divide more efficiently. 96 ÷ 3 Tens Ones 3  90 6 3 	 Finally move into decimal places to divide the total accurately.  TEACH LONG DIVISION ALONGSIDE SHORT DIVISION – see page 40 for examples.

Division - commonly seen in Year 6

Vocabulary share, share equally, one each, two each…, group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, product, division facts,inverse, derive, formal written method.

Abstract representations

```
   h t o
   0 4 1 R1
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
    th h t o
    0 4 0 0 R7
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

```
    h t o
    0 6 1
4 ) 2 4 7
    - 4
      3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
    th h t o
    0 4 0 2
4 ) 1 6 0 9
    - 8
      1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Minimal Resources required to support the CPA approach (depending on year group):

- 10 frames (including egg boxes)
- Straws/pipe cleaners
- Bead strings (to 20 and 100)
- Rekenrek frames
- Abacus
- Numicon
- Base 10/Dienes (including magnetic to model on flip chart)
- Place value grids
- Double-sided counters
- Part-part whole templates
- Place value counters (KS2)
- Multi-link cube